How to close the skill gap?

Parental Background and Children's Skill Development in Indonesia

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Abstract

Preexisting inequalities in socioeconomic status can drive differences in children's cognitive skill development and parents' reactions to child development policies influencing policy effectiveness. To analyze the role of parental background and investments (nutrition diversity and schooling expenditure) in this process, I estimate a dynamic structural model using data from Indonesia. I find two main factors contribute to the adult skill gap: household income and parental education, which influences the productivity of investments. Using the model, I simulate three policies: unconditional cash transfers, nutrition, and schooling price subsidies. To compare their long-run effects on adult skills, I account for parents adjusting their investment behavior in response to policies. Given the same cost, a) subsidizing food prices is more effective than subsidizing schooling expenditure, and b) both are more effective than cash transfers. As I find nutrition and schooling to be complements, a price decrease incentivizes parents to increase both inputs. With cash transfers, parents also increase investments but increase consumption relatively more as price incentives do not change. Nutrition subsidies reduce inequality most effectively, as parents with lower education react stronger to food price changes and, consequently, increase child investments more than parents with higher education. They do so as they spend a larger share of investments on nutrition. Further, nutrition subsidies implemented alone are more cost-effective than any combination of the three policies.

Keywords: *Child development, Human capital, Poverty, Inequality, Cognitive skills, Human capital production functions, Development, Cash transfers, Indonesia*

JEL Codes: *D13, I24, I25, I38, J13, J24, O15*

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1 Introduction

Two-thirds of children globally do not obtain basic skills, and a vast majority of them reside in low- and middle-income countries (Gust et al., 2022).[1] Within these countries, low cognitive skills are concentrated among children from poorer backgrounds. Early in life, they display lower skill levels than children from wealthier households, which translates into a persistent adult skill gap. This gap results in lower intergenerational mobility and higher inequality (Attanasio et al. (2020b)). Simultaneously, there exist significant disparities in parental investments by socioeconomic background. In Indonesia, parents with high school education spend on average more than triple in their child's schooling and invest 15% more in nutrition diversity than parents with no education - who earn less than twice of their income.[2] How much of the adult skill gap is driven by these investment differences compared to parental characteristics? Why does investment behavior vary by socioeconomic status? Are some parents more productive in investing or less resource constrained? Answering these questions is crucial to design effective policies to reduce the gap in adult skills and increase overall skill levels. Different investment behavior by socioeconomic status might lead to parents reacting differently to policies. If so, policies will vary in the degree to which they reduce inequality in skills. Knowing why and when parents invest differently allows to take their response to policies into account and assess the long-run effects of policies on skill levels and inequality.

Therefore, in this paper, I explicitly model parental investment choices and examine how cognitive skill differences transmit from childhood to adulthood outcomes in the setting of Indonesia. Using a dynamic structural model, I quantify the role of parental background and investments (nutrition diversity and schooling expenditure) in skill development. I extend existing frameworks for child development, as Del Boca et al. (2014) and Caucutt et al. (2020), by quantifying the impact of parental decisions on nutrition diversity in children's cognitive development. In doing so, I adapt the framework to a developing country setting. Here, resources are scarce, and food insecurity plays a prominent role in child development (Aurino et al. (2019), Galasso

[1] Basic skills are equivalent to PISA Level 1 skills (able to identify information and carry out routine procedures according to direct instructions in explicit situations).

[2] Author's calculations with data from the Indonesian Family Life Survey (IFLS), supplied by the RAND cooperation. For details, see Frankenberg and Karoly (1995), Frankenberg and Thomas (2000), Strauss et al. (2004), Strauss et al. (2009) and Strauss et al. (2016). Nutrition diversity is measured as the number of food groups consumed.

et al. (2019)). While Attanasio et al. (2020a) and Attanasio et al. (2020b) estimate children's skill formation in a developing country setting, they do not explicitly model parental choices following Cunha et al. (2010). By modeling parental choices, I can evaluate policies' long-run effects, carefully controlling for parental responses. I focus on evaluating cash transfers, food and schooling price subsidies, and their joint implementation. For a careful evaluation of these policies, it is crucial that I estimate the substitutability of schooling and nutrition inputs. The degree of substitutability determines how parents increase investment inputs given price subsidies or budget increases and how much cognitive skills increase in the long run.

I employ and estimate a dynamic structural model where parents face a trade-off between consumption, saving, and investing in their child's skills and are constrained by their income and assets.[3] Parents' socioeconomic background shapes their choices via three key mechanisms, and I incorporate them to differ in influence by childhood period. First, preferences for cognitive skills are allowed to vary by parental education. Parents with lower education might value cognitive skills more as they wish their children to have a better life than them. Second, parental choices are constrained by income and assets, which differ by parental education level. Third, I allow for differences in the technology of skill production. Parents with higher education might be more productive in converting the same level of investments into future skills because they can, for instance, encourage learning during playing. They also might be more productive with schooling expenditure by, for example, being able to support their children with homework. These productivity advantages would allow some parents to invest less and yield the same outcome as parents who invest more.

Using this framework, I estimate children's skill formation for each childhood period. I exploit a rich panel data set, the Indonesian Family Life Survey (IFLS). The IFLS follows a large sample of children over time, recording several measures for cognitive skills and parents' investment choices and characteristics. This feature allows me to account for the time-varying impact of parental characteristics and parenting skills and identify production technology and preferences. Further, I identify if parental investments, nutrition diversity, and schooling expenditure are substitutes or

[3] Different to Del Boca et al. (2014) or Caucutt et al. (2020), I do not model the time parents spend with their children but focus on schooling and introduce nutrition diversity to the model. I focus my analysis on the later periods of childhood as Del Boca et al. (2014) find time to matter less than in early childhood. This might be extended for the evaluation of cash transfers as parental time allocation is highly sensitive to participation in transfer programs (Flores, 2021).

complements using available time and regional variation in food prices. If substitutes, parents increase the demand for inputs which drop in price and substitute the other. However, if inputs are complements, a price decrease in food increases both inputs. This mechanism influences how parents react to policies and their effectiveness. Hence, I can use the model in simulations to quantify the drivers of the adult skill gap and the long-run effects of policies.

My analysis reveals that parents' investment choices are constrained by income and assets, and the closing this gap would reduce the adult skill gap of 0.35 standard deviations (SD) by 0.20 SD. In contrast, differences in socioeconomic background by preferences for children's cognitive skills do not widen the skill gap. Parents with lower education value their children's skills more than their higher-educated peers. Without these differences, the skill gap would be 0.14 SD larger. However, parents, especially mothers, with higher education are more productive in producing cognitive skills.[4] Eliminating these differences would reduce the skill gap by 0.29 SD.

Next, I target the lowest 20% of the income distribution in my policy experiments as income plays a significant role in the skill gap. My simulations show that subsidizing schooling or nutrition prices is more effective than unconditional cash transfers for the same costs.[5] Food price subsidies increase adult skills on average by 0.04 SD and a schooling subsidy by 0.03 SD, while cash transfers have negligible effects. While cash transfers help to lift income constraints, price subsidies change the proportion of investment inputs. As I find nutrition and schooling to be complements, lowering one input price leads to an increase in both inputs.[6] If I compare impacts across the income distribution, cash transfers and nutrition subsidies' impacts decrease with income, while schooling impacts slightly increase. This pattern indicates that parents with low income are significantly more budget constrained and less effective at using schooling investments productively compared to nutrition investments. They spend a higher share of their investment on nutrition resulting in them reacting stronger to nutrition subsidies. Hence, to reduce inequality, nutrition subsidies are the most cost-effective policy. They are also more cost-effective than combining different policies.

[4] Mothers with high school education increase their children's future skills by 20-25% each period compared to mothers with no schooling - holding investment levels and all other factors fixed. Father's education impact equals to a around 10% increase.

[5] Cash transfer size corresponds to 3% of the mean annual income of the lowest 20% of the income distribution.

[6] The percent increase of the targeted input is higher than of the other input. However, the other input increases as well, and therefore total investments.

Further, I find the complementarity of nutrition and schooling to be stronger in high school, resulting in more significant price reactions by parents in this period and higher investment increases. Additionally, cognitive skills show a low persistence. Thus, the impacts in primary school fade out to some extent until adulthood, leading to interventions in high school being more cost-effective than in earlier ages.[7]

Related Literature I contribute to the literature in a three-fold way. First, I add to the research on nutrition and its importance for child development by modeling nutrition diversity as a separate investment input. Doing so, I compare policies accounting for parental responses and identify changes in nutrition and schooling investments due to food price changes. Interventions like food stamp allocation, nutrition supplementation, and cash transfers reduce stunting (extremely low height-by-age), and early childhood stunting has been shown to decrease cognitive skills (Sánchez (2017), Bailey et al. (2020), Galasso et al. (2019), Carneiro et al. (2021)). Nutrition diversity has long run-effects, as early childhood interventions increasing protein intake have been found to result in higher adult cognitive skills (Hoddinott et al. (2008), Behrman et al. (2020)). However, nutrition affects outcomes not only early in life. School meal programs show significant effects for poorer children on test scores in middle childhood (Aurino et al. (2020), Frisvold (2015)). Impacts increase if school meals are designed to be healthy, emphasizing the importance of diversity (Belot and James, 2011). Further evidence shows that children are negatively affected by higher food prices, especially protein price increases (see Vellakkal et al. (2015), Kandpal et al. (2016), Filmer et al. (2021) and Headey et al. (2018)).[8] My results complement these findings as parents increase nutrition diversity with lower food prices leading to higher cognitive skills. However, I depart from the literature by analyzing the co-movement of nutrition and schooling investments. I find schooling expenditure also increases, magnifying food price subsidies' effects.

Second, I contribute to the literature on long-run policy evaluations in developing countries by comparing policies taking into account parental responses. Summarizing

[7] Note that impacts are only evaluated for cognitive skill outcomes. For example, cash transfers might be invested in consumption or to insure against shocks. In my setting, they seem to be effective in lifting the budget constraint for the ultra-poor, as the effect size is double for the most disadvantaged in the targeted group.

[8] Kandpal et al. (2016) and Filmer et al. (2021) show that by a cash transfer in the Philippines stunting decreases via higher protein intake. In comparison, ineligible children are negatively affected in regions with higher protein prices (an association also found by Headey et al. (2018) for protein prices and Vellakkal et al. (2015) for food prices in general).

the existing evidence, Bouguen et al. (2019) conclude that direct investments in health, cognitive stimulation in early childhood, scholarships, and in some cases, conditional cash transfers have positive effects.[9] My contribution lies in simulating the different combinations and synergies of a collection of policies at different points in childhood. By this, I add to the literature on the use of structural models evaluating child development policies (Todd and Wolpin (2006), Duflo (2012), Daruich (2018), Bobba et al. (2021)). I extend this literature by looking, in particular, at reactions to policies subsidizing investment prices. Food price subsidies have been found to have mixed effects on nutrition diversity. Jensen and Miller (2018) do not find any increases for a staple subsidy in China. In contrast, Kaul (2018) and Krishnamurthy et al. (2017) find increases in nutritional diversity, especially of young children, for a price subsidy in India. I extend the literature by modeling several dimensions of parental investment responses to price changes. Additionally, I can focus on the long-run effects on cognitive skills as I estimate skill formation up to adulthood. This feature allows me to model the 'missing middle years' of childhood, primary education, a period which is less researched (Almond et al., 2018). How skill changes by policies translate into middle childhood and how these indicators predict adult outcomes would help compare early life interventions with adolescent ones.

Third, I use data from a lower middle-income country to estimate skill production functions. Parents in low and middle-income countries operate under stronger income constraints, and food scarcity plays a bigger role than in high-income countries. Most of the existing literature on estimating skill production functions uses data from high-income countries (Todd and Wolpin (2007), Bernal (2008), Cunha and Heckman (2008), Cunha et al. (2010), Del Boca et al. (2014), Lee and Seshadri (2019), Caucutt et al. (2020)). Exceptions are, Villa (2017) for the Philippines, Attanasio et al. (2020b) for India and Attanasio et al. (2020a) for Colombia. However, these studies pool investments and do not model inputs like nutrition separately. Thus, parental choices are not modeled explicitly, and their behavior adaptations to policies cannot be simulated. By modeling nutrition and schooling decisions, I can account for

[9] The evidence for the effects of cash transfers on adult outcomes is mixed (see Molina Millán et al. (2019) for a summary). Particularly, for unconditional cash transfers, the long-term evidence is scarce due to fewer trials available (exceptions are Araújo et al. (2018) and Baird et al. (2019)). For Indonesia, Cahyadi et al. (2020) find long-term effects on schooling by a cash transfer program. My model aligns with this finding, as parents increase schooling investments when receiving cash transfers.

parents' responses to policy changes in the simulations and quantify the impact of nutrition diversity on child development in a low- and middle-income country context. Methodologically related to my work are the papers of Del Boca et al. (2014) and Caucutt et al. (2020), as I also explicitly model investment choices. While I use similar methods to estimate parameters, I deviate from their framework by using a different investment input (nutrition), modeling outcomes including adult skills, and using data from a lower-middle income country.

Given the lower-middle income country setting, intra-household allocation and investment trade-offs between siblings can play a role in child development. Calvi (2020) and Brown et al. (2021) find household poverty to be shared unequally between household members. I control for household size and amount of siblings in the estimation and use food diversity, not quantities, which might be more impacted by unequal sharing. Another potential explanation for the skill gap could be imperfect knowledge of skill formation and the child's current skill level. This imperfect knowledge is unequally distributed across parents via socioeconomic status (see Dizon-Ross (2019) and Cunha et al. (2020)).[10] I argue that in my model's context, these differences would lead to underestimating preferences for lower-educated parents (see section 5 for details). Therefore, I treat my estimates as a lower bound. As I estimate lower-educated parents value skills more than higher-educated peers, this gap might be even bigger with knowledge differences. However, extending the framework in this dimension is a promising path for future research.

The rest of the paper is organized as follows. In section 2, I discuss the data used and present facts on the skill gradient in Indonesia. Next, I introduce the theoretical model and describe the estimation procedure in sections 3 and 4. In section 5, I discuss results, which are used in the following two sections to quantify the different contributors to the skill gap (section 6) and simulate policy experiments (section 7). I summarize remarks on results, their interpretation, and ideas for future research in section 8.

[10] Parents with lower education are found to overestimate their children's skills and the impact of their investments compared to their peers. They also tend to underestimate the importance of early life investments driven by the persistence of current skills.

2 Data and evidence on socio-economic background and skills

To motivate model assumptions and the empirical analysis, I start by documenting the skill gap by children's socioeconomic background in Indonesia in subsection 2.2. Using data, I will explore the potential drivers of this gap. However, before discussing the facts in detail, I shortly describe the data I use in section 2.1. For further details on the data, see section A.1.

2.1 Data

As the main data source, I use the Indonesian Family Life Survey (IFLS)[11]. This survey is a panel dataset from 1993 to 2014, allowing me to observe children from childhood to adulthood. Survey waves are 1993, 1997, 2000, 2007 and 2014. The survey area covered represents 83% of the Indonesian population, which gives me regional variation to exploit. The majority of regions not covered in the survey are in the Eastern provinces, which are very remote and poor. The available sample thus allows me to model choices in a setting where investment choices occur as markets are available and schooling options are not strongly limited by availability.

As I model the skill gap between children from different socioeconomic backgrounds, detailed information on the household and investments in children and their skills is necessary. The data set provides information about investments like schooling and nutrition. It follows children long enough to measure materialized skills in adulthood (low attrition rates around 90% to 95% depending on the survey wave). I use survey waves 1997, 2000, 2007 and 2014. I do not use 1993 due to the lack of availability of food prices. Unfortunately, the gaps between waves do not allow me to model the skill process yearly but only in childhood periods (for details, see sections 3 and 4). However, for surveyed years, the panel entails rich information on the household and its members. The household head is the source of the primary data. Interviews also occur with the spouse; more detailed information is collected on 2-3 randomly selected children in the household. My sample for the analysis consists of children for whom information on investments and skills is available. Additionally, they need to have

[11] IFLS data was supplied by the RAND cooperation, for details see: Frankenberg and Karoly (1995), Frankenberg and Thomas (2000), Strauss et al. (2004), Strauss et al. (2009), Strauss et al. (2016) and https://www.rand.org/well-being/social-and-behavioral-policy/data/FLS/IFLS.html

sufficient information on their parents' characteristics. For the estimation, this gives me around 4,563 children in early childhood, 6,329 in primary school and 8,451 in high school (see A.2). Investments used are education investments, like schooling fees, exam fees, books, and health investments. For the latter, I take nutrition diversity as a proxy. Food prices vary by municipality level (kabupaten). In the next paragraphs, I will shortly describe the procedure of constructing price and investment data for each investment input. For further details see section A.1.

For nutrition investments, I use the food consumption information of the household. With that, I can measure which food groups the family consumes. I assume the child to eat from all parts recorded in household consumption. Following Attanasio et al. (2020b), this serves as a proxy for the parents' decisions to invest in the child's health. The food groups counted are vegetables, fruits, dairy, proteins and carbohydrates. Regarding the price of investment for nutrition, I use price data derived from market surveys of the community questionnaires. I use spending reported on schooling fees and materials bought as schooling investments. The price for education, I assume, is one so that the total expenditure on education enters the investments. I only observe schooling investments for primary and high school.

In terms of skill measures, measures for health and cognitive skills are available. For cognitive skills, I use the survey's math, logic or language tests for each child, which I standardize by age and year. In terms of health, I use height and weight, transformed to height/weight-for-age with the help of the WHO Child Growth Standards and WHO Reference 2007 composite data files (Vidmar et al., 2013).

The survey also records other observable characteristics such as the number of siblings, household income, assets and wages. As parental education, I use the parents' education level at the start of the child's life. Thus it does not vary over time. An overview of the descriptives is displayed in table A.1 for children from my sample. One can observe that a fraction of 0.34 is stunting (extremely low height-for age), while 0.09 is wasting (extremely low weight-for age). The fraction of stunting children highlights the food security situation in Indonesia. With the above-mentioned WHO scale for z-scores, children below a height-for-age score of -2 are stunting. Wasting are children below a weight-for-height score of -2 and underweight children below a weight-for-age score of -2. Maternal education is, on average lower than paternal education (years of education). Parents' age varies substantially and is likely not always correctly recorded; however, it does not enter the model except for the household income estimation. A fraction of 0.12 of the sample is not declaring their religion to be Islam, and the gap in household income is wide. Average households have around four adults and two children.

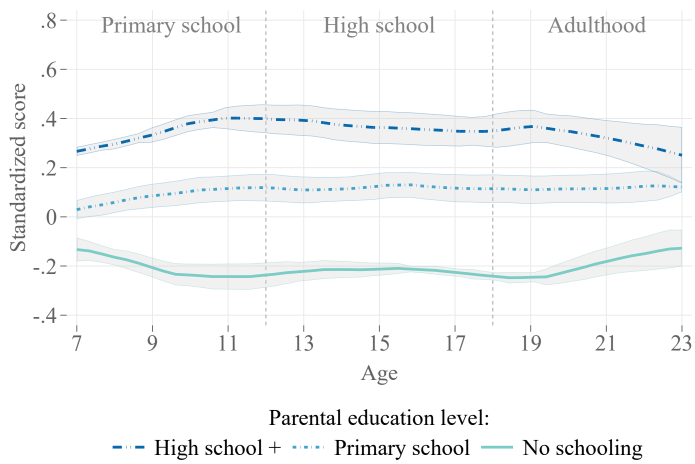
2.2 Empirical evidence on socioeconomic background and skills

Firstly, in this section, I document the size of the skill gap for cognitive skills and health by age in Indonesia. Then, I summarize potential drivers for the skill gap and show how these vary for children from different socioeconomic backgrounds in Indonesia. Last, I will show some descriptive evidence to motivate the need for controlling for unobserved parenting skills.

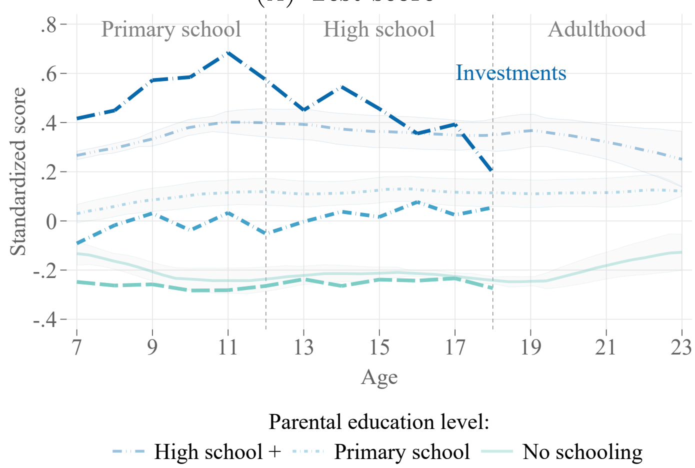
The skill gap in Indonesia is substantial and opens early in life. To show that, I plot averages of skills by parental education group and age in figure 1 and 9. I use standardized test scores for cognitive skills and height to measure health. Visibly, children with lower educated parents show a lower level of health from the start of life (see figure 9a). I only observe test scores from the age of 7, but this initial gap is also large, as shown in figure 1a. For both measures, the gap widens during primary education and closes partly during adolescence. However, it is fairly stable. In adulthood, children from lower educated parents still have substantially lower skills, health and cognitive than their peers. Similar patterns emerge by parental income (see figure 10).

Looking at these differences, the question arises of how this gap interplays with parental investments. To answer this, I plot standardized investments for health; food groups consumed onto the skill gap plot with height in figure 9b. For cognitive investments, I plot standardized schooling expenditure on the graph with test scores (see figure 1b). We can observe a similar gap for cognitive investments. However, the gap widens more in primary school and closes quicker in high schools than the observed skill gap. In contrast, food investment differences are stable over childhood. Thus, parents with higher education mainly increase investments at the end of primary school, while nutrition differences persist over time.

These investment differences are one potential driver for the skill gap and can be driven by several mechanisms via which parental education influences children's skills. Foremost, parents with lower education have fewer resources to invest in their children.



(A) Test score



(B) Test score and investments

FIGURE 1: Children skills and investments over age by parental education

Note: Skills are fitted with local mean smoothing by age and parental education groups. Parental education groups correspond to the average education of both parents. Confidence intervals displayed are at 95% level. Investments plotted are standardized schooling expenditures. Scores of skills and investments are standardized by age to have a mean of 0 and SD of 1.

As shown in table 2.1, lower educated parents have less income available. By that, they can invest less in children, both for nutritional investments and for schooling. Differences in investments are substantial; parents with high school education spend more than triple on education than their counterparts without education.

Income is not the only potential source of the gap between children's skills. Parents with lower income and education might have lower cognitive skills and worse health. On the one hand, this might lead to different initial skills for the children, which I observe in the data. However, their abilities and health might influence their investment

Table 2.1: Potential sources for the skill gap by maternal education

	Parental education level:					
	None	Primary school	High school	F-test	Mean	Sd
Resources						
HH income	181.02	384.53	522.77	0.00	289.19	479.74
Maternal skill set						
Test score	-0.44	0.24	0.51	0.00	-0.00	1.00
Height	-0.15	0.13	0.31	0.00	0.00	1.00
Initial skill levels						
Test score	-0.23	0.21	0.37	0.00	-0.00	1.00
Height-for-age	-0.17	0.18	0.41	0.00	0.00	1.00
Childhood investments						
Food groups consumed	3.36	3.71	3.85	0.00	3.57	0.91
Education spending	2.30	5.37	7.53	0.00	5.14	10.50

Note: The last column displays p-values for the null hypothesis that means for none and high school education are equal. Skills are normalized to 0 mean, SD of 1. All values are from period 2 (age 6-11), except initial height. Income and education spending expressed in 100,000 rupees.

productivity. Parents with higher abilities might be more capable of helping children with homework, which makes their schooling investment more productive.

Apart, parents with higher education seem to invest differently. They spend more significant amounts of money on education. Despite income differences, this might be driven by differences in productivity, similar to the productivity differences by ability mentioned above. Also, parents' preferences might vary with education. Higher educated parents might differ in valuing skills to their peers. For instance, lower-educated parents might wish for their children to do better off than them and invest more. However, resources might constrain them in doing so. As visualized in figure 2, households in the lower part of the income distribution, spend a significant larger part of their income on investments in their child. This indicates their income constraint but might also be an indicator for stronger preferences for skills.

I can only uncover these mechanisms in a structural model, not with the descriptive data available. Therefore, I construct a model where parents decide on different investment inputs, which productivity varies by parental education, among other factors (described in further detail in section 3). These parents face income constraints and value child skills differently by education.

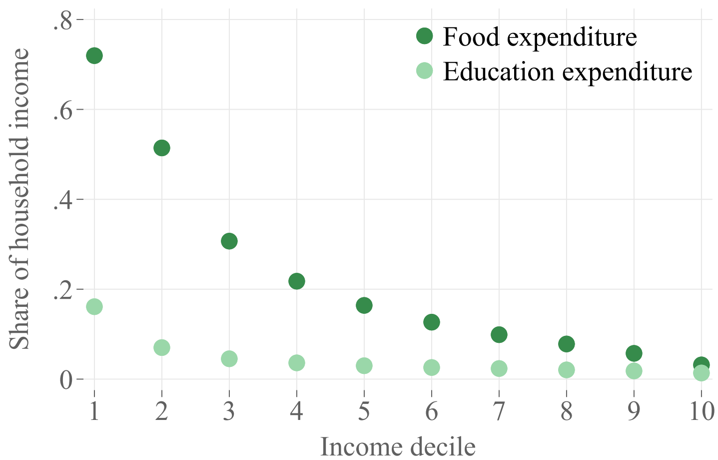


FIGURE 2: Fraction of household income spend on child investments
Note: Expenditures shares are plotted as median fraction of total household income by income decile. Household income is adjusted by household size.

However, controlling only for observable characteristics of the parents might miss an important feature: parenting skills. Some parents could have higher parenting skills, leading them to make better investment decisions due to higher ability. If I omit to control for those that correlate with education, it will lead to biased estimates. To illustrate that they are not aligning with education and income, I plot distribution by parent's income and education groups on figure 11.

As one can see, the distribution in the lower education and income categories is skewed to the left. However, even in these categories, there is substantial heterogeneity, which parenting skills can drive. The impact of these skills might vary by childhood period, similar to the impact of other potential drivers of the skill gap. Resources might play a more critical role during high school than in early childhood since higher investments are needed to affect future skills.

The potential drivers call for a model-set up where investment effects vary across periods and controls for unobserved parenting skills. Also, skills effects on the following period skills need to change over time. Including these dynamics in a theoretical model might allow policy simulations to mimic the potential fade-out of interventions and to see when and why this happens.

3 Model

To capture the empirical facts described in section 2, the theoretical model entails different channels via which socioeconomic background influences skill development. Thus, it captures investment decisions influenced by education and features households' budgets to constrain investment expenditure. Additionally, I will account for parenting skills in the skill production function, and all these influences vary by childhood period.

Regarding modelling choices and functional form assumptions on the skill production function, I follow Del Boca et al. (2014) and Caucutt et al. (2020). However, in contrast to both, I focus on nutrition and schooling inputs instead of time inputs. Hence, this model will especially capture later childhood periods, where monetary expenditures become more productive and feature the transition of skills in teenage years to adulthood.

Households represent a parent-child pair. Parents decide on investments into the child each childhood period (early childhood, primary school and high school). In the final period of the model, the child grows up to be an adult, and no further decisions take place. In the decision periods before the child becomes an adult, households derive utility from consumption c_t and current child's skills Ψ_t. In the final period, households only derive utility from the final skills of their child Ψ_{T+1} and assets a_{T+1}. The latter is merely to assure that parents do not deplete assets fully in the high school period to maximize utility in the last period.

To optimize their utility, parents decide to invest their resources into consumption c_t, savings a_{t+1} or investments in the child I_t. Hereby, parents are constrained by their income and their decisions are influenced by the prices of investments. I adjust household income by household size (see A.1 for details). For the moment, I abstain from further modelling the trade-off in investing between siblings, which would be a potential future extension of this model. Further, as the model only contains monetary investments into children, time does not play a role in the skill production. The trade-off between time at home might only be with consumption and not with spending time and investing it in the child. For this reason, I do not model labor choices as the trade-off between consumption and leisure is not the focus of the model.

Investment decisions are made every period to be able to measure when they matter the most for skill development. Figure 3 illustrates a graphic overview of the timeline. Periods are determined by the child's age, following standard definitions in the literature for an early childhood period, primary education and secondary education. In period $t = 0$, the child is born with an initial skill endowment Ψ_1; then, in early childhood, the household decides on nutrition n_t. In later periods, the parents also choose how much to invest in schooling s_t. In $t = 4$, the child is grown up, and final cognitive skills outcomes realized.

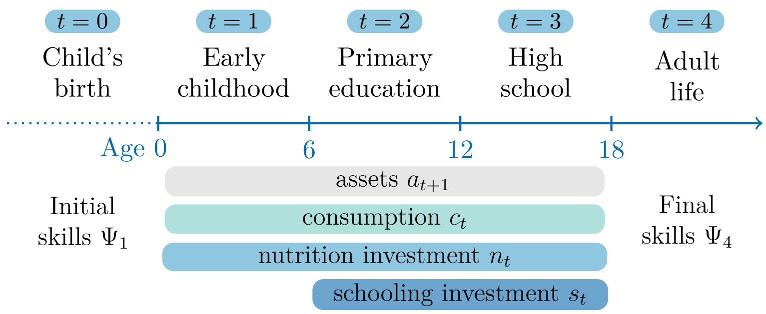


FIGURE 3: Model stages

Formally, each period the household maximization problem looks like the following:

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, n_t, s_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\ & + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad c_t + p_{n,t}n_t + p_{s,t}s_t + a_{t+1} &= (1+r)a_t + y_t \\ a_{t+1} &\geq a_{min,t} \end{aligned} \tag{1}$$

Households maximize utility with respect to consumption c_t, assets a_{t+1} and investment choices. Investments in the child are investment in nutrition n_t and an schooling investment s_t in period 2 and 3 ($s_t = 0$ in period 1). Nutrition investment can be understood as a proxy for health investments and is measured by the number of food groups a child consumes. Therefore, this measure is a food diversity measure and does not capture food quantity. All investments are associated with their corresponding prices in the budget constraints. The price for nutrition is $p_{n,t}$, and the price for one unit of schooling is $p_{s,t}$. The vector of all prices for investments is denoted by Π_t. The household cannot spend more than their current income y_t and assets a_t. Future utility depends on the evolving state space of future income and prices, as well as future household characteristics Z_{t+1} and future skills Ψ_{t+1}. Households can borrow, but not more than $a_{min,t}$, the maximum amount a household can be in debt.

The current period's utility depends on consumption and skills. The utility functions take the corresponding forms:

$$u(c_t) = \ln(c_t) \tag{2}$$

$$v(\Psi_t) = \ln(\Psi_t) \tag{3}$$

In the last period of the model, utility exclusively depends on the final skill level of the child Ψ_{T+1} and final assets. By that, a motivation to invest in the child is ensured. Also, not all assets are depleted in the last period:

$$V_{T+1} = u(\Psi_{T+1}) = \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \tag{4}$$

Here it is important to note that the altruism factors α_e and γ_e depend on parental education. By this, I allow parents to value their child's skills differently depending on their education. In the adult period, no decisions take place, so the child's skill level is the only variable from which the household derives utility apart from accumulated assets.

What is left to specify is how children's skills evolve. Future skills will depend on current investments I_t, current skills Ψ_t and a total factor productivity $\theta_t(Z_{\theta,t})$:

$$\Psi_{t+1} = \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \tag{5}$$

Thus, $\delta_{1,t}$ will describe the impact investments have on future skills, which varies by period. The self-productivity of skills Ψ_t is expressed by $\delta_{2,t}$, also varying by period. I ensure that the estimation is flexible enough to capture that early childhood skills might be not as critical for future skills than skills in high school. Persistence of skills is likely to increase over childhood, and this functional form allows to capture this development flexibly. The total factor productivity depends on observable characteristics $Z_{\theta,t}$. These are parental education and the age of the child.

Total investment are composed of the investment inputs nutrition n_t and schooling s_t:

$$I_t = [n_t^{\rho_t} + a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t}]^{\frac{1}{\rho_t}} \tag{6}$$

I assume a CES investment function, following Caucutt et al. (2020). The parameter ρ_t describes the elasticity of substitution between nutrition and schooling. Schooling investments have a relative productivity of $a_{s,t}$, which depends on observable characteristics. These are parental education e, age, number of siblings and the unobserved parenting skills η. Productivity depends on parental education since one could imagine that the investments have differential effects by parents' education. Higher-educated parents might buy adequate books for schooling when the child needs them or be able to help the child with homework at later levels of schooling. In a similar spirit, unobserved parenting skills η influence productivity. Controlling for the number of siblings allows either siblings to help with homework or reduce the time parents can spend with the child on homework, thus reducing the productivity of schooling. An assumption is that $a_n = 1$, thus the productivity of nutrition investments is normalized for identification. In early childhood $I_t = n_t$

The elasticity of substitution each period ϵ_t is measured by ρ_t with $\epsilon_t = \frac{1}{1-\rho_t}$. Thus, if $\epsilon_t < 1$ the investments are complements, if $\epsilon_t \geq 1$ they are substitutes. The elasticity will drive price reactions. Suppose goods are substitutes and the price of one rises. In that case, it will be substituted by the other one to some degree. If they are complements, this substitution will not happen, and overall investment might be decreased depending on the degree of complementarity.

Depending on the productivity of each investment, price rises will have different impacts on investments varying by parental education and other observable factors. For instance, if food prices rise and the goods are substitutes, investments might shift to more schooling expenditure. However, if schooling investments are more productive for high-educated mothers, they might have to buy less quantity to substitute for the loss in nutrition. In terms of complements, the substitution would not take place. However, if schooling is more productive for high-educated parents, changes in food prices might impact them less than low-educated parents. This interplay shows why it is essential to know if investments are substitutes or complements. This knowledge can help to suitable policies can be designed. In the case of substitutes, a price subsidy on one product might lead to less investment in another. In case of complements, this might lead to an increase in all types of investment.

As Caucutt et al. (2020), Moschini (2019) and Molnar (2018), I exploit the fact that the maximization problem can be separated into an inter-temporal and an intra-temporal problem. The intra-temporal problem minimizes the costs for investments for a given amount of total investments I_t. The inter-temporal problem will then maximize utility with respect to total investments and consumption. The minimization problem will minimize costs for a given level of investments I_t. The minimization problem takes the following form:

$$\begin{aligned} \min_{n_t, s_t} \quad & p_{n,t}n_t + p_{s,t}s_t \\ \text{s.t. } & It = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} \end{aligned} \tag{7}$$

I can derive solutions for each investment input given the total investment level. With having derived equations for the investment inputs n_t and s_t given I_t, I can reduce the maximization problem to maximizing with respect to I_t, simplifying derivations (see section A.6). Then, the inter-temporal problem can be characterized by:

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, I_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\ & + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad c_t + \Lambda_t I_t + a_{t+1} &= (1+r)a_t + y_t \\ a_{t+1} &\geq a_{min,t} \end{aligned} \tag{8}$$

Λ_t will describe the price for one unit of total investment, which arises from the results of the cost minimization (see section A.6). Given the results, investment input prices will determine the amount of each investment input and the price for one unit of total investment.

Hence, the model captures investment decisions in children influenced by investment prices and parental preferences, differences in investment productivities and parenting skills. I allow for the interplay of the budget constraint, preference parameters and productivity of skill formation differing by education and observables. This way, I can quantify how and when income and parental education influence children's skill development most. Further, it allows me to distinguish between the influence of nutrition and schooling as inputs and when they have the highest impact on skill development.

4 Estimation and calibration

To estimate the model, I take the following steps:

1. Estimation of types of parenting skills by k-means algorithm

2. Estimation and prediction of household income by OLS

3. Estimation of skill formation parameters by joint Generalized Method of Moments (GMM) for:

    - Investment parameter: using relative demand ratio moments
    - Human capital parameters: using skill moments and factor loading moments

4. Estimation of preference parameters by simulated methods of moments (SMM)

In the following paragraphs, I describe each step in the listed order in detail. For further details, see appendix A.3. In step 1, I start the estimation procedure by determining the unobserved parenting skill types. Since all equations depend on the types $k = \{1, ..., K\}$ of unobserved parenting skills η, these need to be estimated first. To do so, I use the k-means algorithm in the spirit of Bonhomme et al. (2022) to control for unobserved heterogeneity. The advantage of this method is that it allows for types whose impacts vary over childhood periods. Additionally, estimating the types outside the model is less computationally intensive, and the strategy uses empirically relevant data to determine the types. For identification, I can exploit the fact that I observe parents over time and across children (siblings) in terms of their investments. Assuming that the impact of parenting skills is the same for each child, I can use this additional data to identify the skill type for each pair of parents.

To perform the k-means algorithm, data moments must be chosen, which are influenced by the types. In my case, these are schooling, nutrition, and household income investments. I assume investments to be partly driven by unobserved parenting skills and that these skills can translate into higher productivity in the labor market resulting in higher income. The moments I calculate are lifetime averages of parental investment decisions and income across childhood periods and their children. I calculate lifetime moments because an assumption of the k-means algorithm is that parents of the same type would converge over the life cycle to have the same moments with $T \to \inf$ (for details, see A.3).

Thus, I can use the variation in lifetime moments in the data to determine types. To do so, the algorithm minimizes the within-cluster (type) variance. The state-space is split into clusters, so that parents within a cluster are as similar as possible:

$$\min_{k \in \{1,..,K\}^N} \sum_{t=1}^{N} \sum_{c=1}^{C} ||\boldsymbol{m_{t,c}} - \overline{\boldsymbol{m}}_{\boldsymbol{k}}||^2 \tag{9}$$

where $\overline{m}_k$ is the average of moment vector m of parenting skill type k and t stands for each period in the data, while c indexes each child the parents have. The moments are standardized to have mean zero and variance one for the k-means algorithm. To run the minimization, the researcher needs to set the total amount of clusters K. With the help of the elbow and silhouette criteria, I determine the optimal amount of types K, as plotted in figure 12. These two criteria determine which amount of clusters decreases the variation within cluster and increases variation between clusters without adding computing time significantly. The optimal number is $K = 4$. A detailed discussion of robustness checks is in the appendix A.3. Then, I can determine for each parent pair the unobserved parenting skill type they have according to the algorithm.

Moving on to step 2, having estimated parenting skills, I use these as inputs to estimate household income with a standard Mincer equation since I abstract from modeling labor choices. Household income depends on parental education, number of household members, rurality, age of the household head, and parenting skills. The parameters for these characteristics will then be used to predict household income for the calibration and simulations. For these predictions, I assume the income shocks to be i.i.d. normally distributed. Thus $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

In step 3 follows the estimation of the human capital and investment parameters consists of a joint GMM estimation. For this estimation, I derive a set of moments for the investment function parameters in equation 6 and another for the human capital parameters in equation 5.

To do so, for the investment parameter moments, I start by deriving and rearranging the first-order conditions of the cost-minimization problem to formulate the following linear relative demand equations, which I can estimate with OLS for periods 2 and 3 (for derivations, see A.6):

$$\ln\left(\frac{p_{n,t}n_t}{p_{s,t}s_t}\right) = \frac{1}{\rho_t - 1} Z_t'\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln\left(\frac{p_{n,t}}{p_{s,t}}\right) - \frac{1}{1-\rho_t}\eta + \epsilon_{ns,t} \tag{10}$$

The relative demand ratio between nutrition and schooling quantities will depend on observable characteristics $Z_{s,t}$. These form, following Caucutt et al. (2020) assumptions, the relative schooling productivity $a_{s,t}(Z_{s,t}, \eta) = exp(Z_{s,t}'\phi_s + \eta)$. Note, as mentioned in section 3, I normalize $a_{n,t}(Z_{n,t}) = 1, \phi_{n,t} = 0$ to identify all parameters. Thus, I will only be able to have results on the relative magnitude in terms of their impact on the productivity of investments. The characteristics $Z_{s,t}$ include paternal and maternal education and other observable characteristics such as religion, age of the child, rural area, siblings in the household, and gender. Additionally, the productivity will depend on η, the unobserved parenting skill type, as one can see in equation 10. $Z_{s,t}$ here is a matrix of variables as parental education. As one can see ρ_t, the substitution parameter for nutrition and schooling is identified with the price ratio of these inputs. As schooling prices are assumed to be 1, this parameter will be identified by variation in the food price.

As instruments $Z_{t,ns}$ for the GMM moments displayed in equation 10, I use the observable characteristics $Z_{s,t}$, the price of inputs and parenting skill types k. Thus I assume the moments to be orthogonal:

$$\mathrm{E}\left(\left[\ln\left(\frac{p_{n,t}n_t}{p_{s,t}s_t}\right) - \frac{1}{\rho_t - 1}Z_t'\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln\left(\frac{p_{n,t}}{p_{s,t}}\right) - \frac{1}{1-\rho_t}\eta\right]Z_{t,ns}\right) = 0 \tag{11}$$

For this equation to be accurate, I need to assume that the measurement error in equation 10 is independently distributed across individuals, and no variables in the error term influence the demand ratio and instruments used for the moment equations. For this not to be true, a variable would need to influence schooling and nutrition inputs differently, as influences of the same magnitude factor out by the ratio. For example, not controlling for parenting skills η might bias the results as it could influence schooling differently from nutrition but be correlated with parental education. It might be driven by ability which influences education and via parenting skills, also the ratio of investment incomes.

To control for this potential bias, I use the estimated types from step 1. As these estimated types do not correlate strongly with education, I assume that education is not working solely through parenting skills in influencing the ratio of nutrition versus schooling parents spend. I understand the influence of education, to be for example, knowing to help your child with homework. In contrast, unobserved parenting skills capture, e.g., parents' empathy to react to their children's problems at school and spend more time with them, which then increases their productivity at school as it might mitigate behaviors that hinder learning.

Note that the identification of the substitution parameter ρ_t depends on food prices, whose variation I assume to be exogenous. Parents' choices might influence food prices or schooling fees, which would break this assumption. For instance Bold et al. (2015) find that providing free public primary education shifted parents demands to private education and increased prices for these schools in Kenya. I do not model differences in public and private education provision and the supply side for simplicity, a caveat to keep in mind for interpreting the results. Regarding food prices, Filmer et al. (2021) find that cash transfers lead to higher food prices for proteins by increased demand of recipients having negative effects on ineligible children. However, these results are mainly found in remote areas or when a large proportion of the village received treatment. In this context, this is unlikely to be the case, as I look at only a subpopulation, relatively urban areas, and not extremely remote villages. Thus, for simplicity, I abstract from modeling prices, but this could be a future extension of the model. Nonetheless, it is vital to keep this simplification in mind when evaluating the outcomes of the policy experiments.

Turning to the human capital parameter moments, I will mainly use equation 5 which describes how current investments and skills translate into future skills. However, one must consider how skills are measured in this context before estimating these parameters. I use logic (raven) and math test scores in the later periods of the model for cognitive skills and height and weight in early childhood as a proxy. These measures, however, only proxy the latent skills and are measured with error. To account for this, I follow Cunha et al. (2010) and assume a measurement system for the latent skills Ψ_t. The system looks like the following:

$$S_{ts_1,t} = \lambda_{ts_1,t} \ln(\Psi_t) + \epsilon_{ts_1,t} \tag{12}$$

and:

$$S_{ts_2,t} = \lambda_{ts_2,t} \ln(\Psi_t) + \epsilon_{ts_2,t} \tag{13}$$

where ts stands for test scores I use in the corresponding period. Following Caucutt et al. (2020), I normalize one factor loading $\lambda_{ts_1} = 1$ each period.

Combining the measurement system equations 12 and 13 with equation 5 for the skill formation process,I derive additional moments for the GMM estimation (for details see A.6):

$$\frac{1}{\lambda_{ts,t+1}} S_{ts,t+1} = \phi_{\theta,t} Z_{\theta,t} + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts,t}} S_{ts} + \epsilon_{\Psi,t} \tag{14}$$

Moreover, to identify the factor shares:

$$0 = \mathrm{E}[(S_{ts_1,t+1} - \lambda_{ts_2,t+1} S_{ts_2,t+1}) S_{ts_1,t}] \tag{15}$$

and:

$$0 = \mathrm{E}[(S_{ts_1,t} S - \lambda_{ts_2,t} S_{ts_2,t}) S_{ts_1,t+1}] \tag{16}$$

In this context, $Z_{\theta,t}$ entails parental education and the child's age. Again, I assume these factors to map into the total factor productivity $\theta_t(Z_{\theta,t})$. As instruments Z_{t,Ψ_t} for the skill moments I use the characteristics in $Z_{\theta,t}$ and investment inputs schooling s_t and n_t. Thus:

$$\mathrm{E}\left(\left[\frac{1}{\lambda_{ts,t+1}} S_{ts,t+1} - \phi_{\theta,t} Z_{\theta,t} + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts,t}} S_{ts}\right] Z_{t,\Psi_t}\right) = 0 \tag{17}$$

I abstract for modeling investments between the points of time I observe the children in the data. I do not have enough information on investments or income to impute those. Another shortcoming is that while I control for measurement error in skills, I do not do so for investments, which could lead to biased results, and therefore the results have to be taken with caution. However, as I do instead treat investments in nutrition as a proxy for health investments and schooling for education investments, these inputs are not supposed to be understood as precisely modeled. In general, measurement error in investments is likely to decrease the coefficient of investments, thus underestimating the impact (Cunha et al., 2021).

After this estimation procedure, I move to step 4 and estimate the preference parameters γ_e, α_e and ζ. To do so, I use the optimal solution for total investments and assets (see section A.6 for details) in the simulated methods of moments. I set the discount factor β to 0.98, following calibrations in the literature on Indonesia (Dutu, 2016). I match mean investments by childhood periods and parental education level and assets by period to their data counterparts (see section A.3 for details). For the SMM and simulations, I assume wages and prices change over time. However, for simplicity, for the transition of state variables, I assume all other household characteristics to be fixed. Thus, households do not move from rural to urban areas, and the number of siblings does not change. This process could also be enriched in future research.

5 Results

I will discuss the results in order of the estimation strategy described in section 4. Thus, I start with the parenting skills types. Remember that in the model, parenting skill types capture unobserved heterogeneity among parents, influencing their investment behavior. I assume there are parents who, independent of income or education, might be more effective in investing in schooling. If these types are more effective in schooling, they will shift their investments to schooling rather than nutrition, which influences schooling and nutrition investment levels. I assume these parenting skills also influence income. A parent with certain parenting skills might be better at communication, increasing their income. I determine types by using the variance in investments and income with the help of the k-means algorithm. The outcomes of the k-means algorithm suggest that there are four types. These types are different in investment levels and income, driven by the identification method. In the upper graph of figure 4 I show the types' distribution and their characteristics in terms of income and investments (see table A.3 for further details). The two most often occurring parenting skill types, 0 and 1, have low income and schooling investments compared to the other types. Additionally, type 1 also has low food investments. In contrast, type 2 has higher income but also very high education expenditure. Type 3 seems to have mainly very high income and modestly increased investments. Types could be, in general, correlated with education. If they are correlated strongly, this will cast doubts on their identification. To check, I show the education distribution in the bottom part of figure 4. Types are partly correlated with education, but there is still substantial variation within education groups. The share of mothers with no schooling is higher for the low-income and low-investment types 0 and 1, while the share of high school mothers is higher for types 2 and 3. The share of mothers with primary education is similar for all types. Hence, while there is some correlation between education and types, there is still some variation regarding unobserved parenting skills within education groups.

Turning to the results on household income, one can observe that these parenting skill types matter. In table A.4, one can see that types 2 and 3, which are associated with higher income (table A.3), also tend to have higher productivity of income in the household income estimation. Especially type 3 has high productivity, which is the one with the highest observed income, while type 1, the lowest, is associated with a negative coefficient. In terms of magnitude, being of type 2 corresponds to

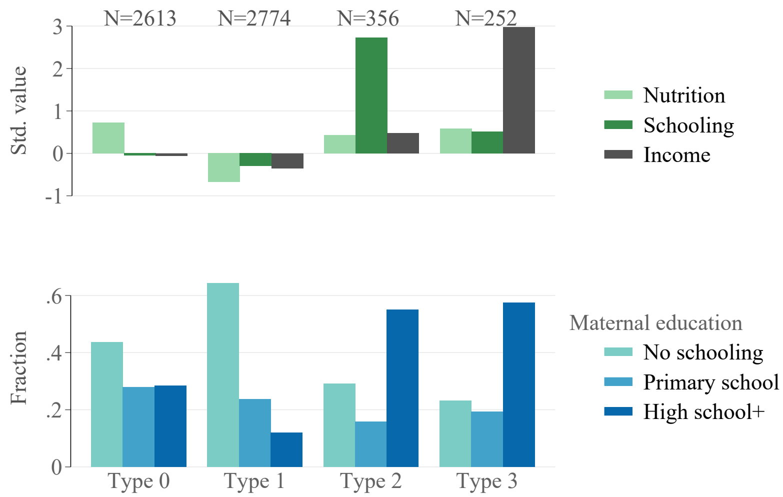


FIGURE 4: Characteristics of parenting types η (investments/resources and education)
Note: Nutrition is food groups consumed, schooling describes schooling expenditure, and income annual household income (lifetime averages by parenting pair).

an increase in household income of having a mother with a high school education. Furthermore, being of type 3 exceeds this by influencing a third more than both parents' high school education. Unobserved parenting types are likely to contribute to the gap by socioeconomic status. They are driven part of the income differences between parents. The other coefficients from the household income estimation show the expected signs and magnitudes; education and age increase income, while living in a rural area decreases it.

The GMM estimation results for investment parameters using equation 10 reveal the degree of complementarity for investment inputs and their productivity by period (see table 5.1 and for further parameters A.5). Nutrition is complementary to schooling in both periods, primary and high school. Consequently, if prices for nutrition increase, parents decrease their investments in nutrition and schooling. Worth to note that the complementarity increases in high school with a higher substitution parameter ρ_t of -11.38 versus -3.75 in primary school. The complementarity is stronger than what Caucutt et al. (2020) find for time and goods investments ranging around -1 for the US.

The higher degree of complementarity in high school leads to parents responding to price changes of one input with decreasing demand for the other one stronger than in primary school. A reason for this reaction might be that in primary school, schooling is mandatory, making the demand for it less elastic. However, in high school,

Table 5.1: Estimation results for investment parameters

	Primary school		High school	
Investment elasticity:				
ρ_t	-3.75	(0.86)***	-11.38	(5.11)**
Implied elasticity	0.21		0.08	
Schooling investment productivity $\phi_{s,t}$:				
Constant	-3.68	(0.51)***	-42.17	(16.55)**
Mother primary	1.10	(0.25)***	3.06	(1.32)**
Mother high	1.87	(0.39)***	5.04	(2.15)**
Father primary	0.09	(0.16)	0.63	(0.47)
Father high	-0.08	(0.19)	0.51	(0.50)
Parenting type 1	-0.24	(0.14)*	0.06	(0.34)
Parenting type 2	4.74	(0.97)***	9.62	(4.10)**
Parenting type 3	1.64	(0.50)***	2.47	(1.29)*
Observations 27,366				

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

parents reduce investments more in their children if food prices increase as securing the households food consumption is a priority and schooling is less mandatory. An additional reason might be that older children can contribute to household income and could have more agency on what income is spent on and might think more myopically about investments. With higher prices, they are needed to sustain the household and at least ensure primary education for younger siblings. For parents, it is only efficient to reallocate investments to the relatively cheaper input schooling even with the consumption constraint. Reallocation does not happen because strong complementarity means that if both investment inputs increase simultaneously, this yields the highest total investment. Increasing only one is not efficient.

Considering policies, this is an essential result since decreasing nutrition prices might increase food diversity and schooling expenditure. However, this depends on how parents react to price changes (e.g., if they reallocate money to another input or spend the money for consumption). For this question, policy counterfactuals are necessary. In general, the complementarity of schooling and nutrition is in line with findings that children's test scores increase with the availability of school meals (see Alderman and Bundy (2012), Chakraborty and Jayaraman (2019) and Aurino et al. (2020)). Nutrition increases learning ability; and further increasing both inputs yields higher skills than increasing only one.

Additionally, schooling productivity differences might affect how parents react to price changes. Regarding productivities, table 5.1 shows results how these vary with parenting type and education. The relative productivity of schooling increases with maternal education, especially in the last childhood period. Thus, schooling is more productive for children with mothers with high school education. Similarly, parenting types 2 and 3 are more productive in schooling. Living in a rural area decreases the productivity of schooling, especially in high school. This magnitude offsets the productivity increase of having a mother with a high school education. Having siblings negatively influences schooling productivity, more so in high school, while not being Muslim increases productivity. By the same magnitude, productivity increases for female children, both are only significant in the high school period. Parents with high productivity will invest a higher share in the more productive input than parents with lower productivity. Other estimation and calibration results are needed to interpret the results on productivities for policy implications because these enter several spots in total investment prices and investment choices.

These parameters mentioned above describe the total investments parents will supply. To link parental investments to skill, table 5.2 displays estimation results from the key parameters in equation 5 which quantifies the impact of parental investments and current skills on future skills. The human capital parameter δ_1 describes the impact current investments have on future skills, δ_2 characterizes the impact of current skills. They are multiplied by the total factor productivity of parents, which varies by their education and the child's age and is characterized by $\phi_{\theta,t}$.

The human capital parameters, δ_1, δ_2, and the factor productivity vary by period. Looking at magnitudes, investments have a higher impact early in life, with a coefficient size of 0.28, and similar impacts in primary and high school with sizes 0.16 and 0.18. These magnitudes can be interpreted as the fraction of a standard deviation increase in test scores if investments increase by one log point. Thus, investments impact the next period's skills more in early childhood than in other periods. Looking at the impact of current skills δ_2, skill persistence increases over life. In the first period, the current skills have a lower impact on future skills (0.1 in magnitude). However, in the first period, I only used a proxy for cognitive skills, which are health measurements. These parameters are not directly comparable and just indicative in their compared magnitudes. In later periods the persistence of skills ranges around 0.2.

Table 5.2: Estimation results for human capital parameters

	Early childhood		Primary school		High school	
Human capital parameters:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.18	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.22	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.11	(0.03)***
Observations	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1.

This persistence is relatively low compared to other findings in the literature. Cunha et al. (2010) find a very high persistence of cognitive skills using US data. However, in India Attanasio et al. (2020b) find a similar low persistence for cognitive skills at age 8 as I do. They find a higher one at age 12. Indicated by the lower persistence in India and Indonesia than in the US, noisier skills measures might also drive this. The US data uses age-adjusted test scores, which are comparable between waves. They might more accurately display skills. I account for this measurement error but assume that errors are not correlated. Therefore, future work is needed to account for measurement error under weaker assumptions and using data with more precise measures to support the analysis in this paper. In terms of investments, I find higher impacts than Caucutt et al. (2020) and Attanasio et al. (2020b). However, these coefficients are harder to compare due to different investment inputs and functional form assumptions. Nonetheless, the findings of Bailey et al. (2017) speak for a lower persistence of cognitive skills, at least when measured in test scores and not underlying intelligence. In the meta-analysis of early childhood interventions, a significant amount of them display fading out effects on cognitive skills.

To illustrate the magnitudes, I compute the effect of rising current skills and investments by one unit on future skills. The calculations are visualized in figure 5 for each childhood period. I take average skills and investments as base comparisons for the main calculation. To illustrate what increases of one unit mean for children with low investments, I also calculate the percentage increase for base investments of one.

This increase in comparison to current investments of one is higher than in the case of three, leading to a higher growth rate. This is relevant for policies, as it means that for the same costs of one unit of investments, increasing them for the children with low investments will lead to large increases. Adding one unit of investments increases future skills by around 9% in period one and around 5% in later childhood periods. In comparison, from a lower level of investments, adding one unit induces an increase of 20% in the first period and around 12% afterward. In contrast, adding one unit of skills to the current skills in early childhood leads to 6% higher skills in primary skills. Later, the effect of increasing skills by one unit is higher than that of investments, increasing to around 12-15%. Thus, investing early to increase current skills in the next period leads to higher adult skills with lower costs.

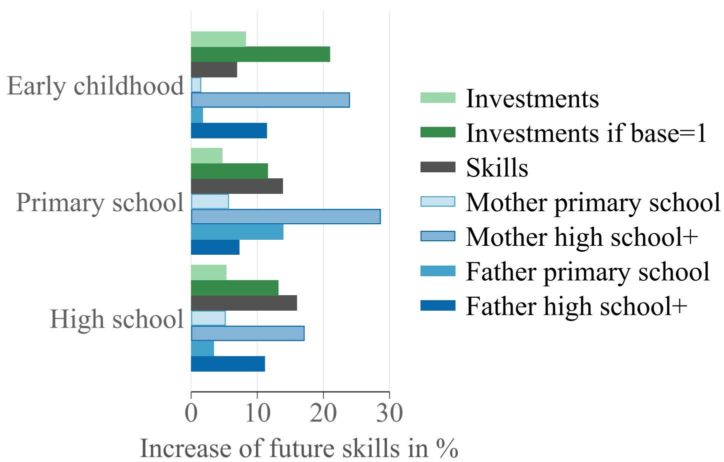


Figure 5: Increase of future skills if characteristic/input increases by one unit

Note: Percent increase of future skills if investment or skills increase by one unit. Increases calculated with sample means as base skills (1.01) and base investments (3) if not otherwise indicated. For parental education, the base category for calculation of changes are parents with no schooling.

The total factor productivity (TFP) increases the impact skills, and investments have, as it multiplies with these values. This productivity might vary with parental education. Results in table 5.2 show that in early childhood, only parents with high school education have a higher TFP, whereas, in later periods, also parents with primary school education do so. While maternal education's impact decreases over childhood, paternal education seems to stay the same in magnitude. The impact of age is negligible. The coefficient sizes translate into percentage differences in the following period skills as depicted in figure 5. Having a mother with a high school education leads to around 25% higher next-period skills in early childhood and primary school and 18% in adulthood. Father's education, in contrast, has a lower impact, around 10%. These differences also magnify investment or skill input changes as they multiply with skills and investments in the skill formation equation (see equation 5). A reason for these high magnitudes could be neighborhood effects, as I only control for rural areas but not more nuanced units (see Chetty and Hendren (2018a), Chetty and Hendren (2018b)). Parents with high school education might live in districts with better amenities or schools. Similarly, they might send their children to different schools. If the qualities of these schools are not reflected in the differences in fees, I do not capture them separately but with the productivity differences by education. Also, as Biasi (2021) shows, differences by the district in public school financing influence intergenerational mobility, which is not necessarily reflected in fees. Another explanation might be that higher educated parents play with their children and might have access to toys that encourage learning. Therefore their children are better at accumulating human capital. Further, this might also be an inherited ability. More nuanced and detailed analyses and a different model are needed to disentangle these potential effects further. Therefore I abstain from framing these further and leave this to future research. In general, the magnitudes of technological differences highlight that even if parents with less education invest the same in their child as a parent with high education, their returns will be lower.

With this set of parameters, one can now calculate how each period's investment differences translate into differences in adult skills. In A.9, I calculate what impact closing differences in investment inputs has on adult skills holding all other factors constant. The differences are between parents with high school education and parents without schooling. I calculate what closing the gap in this period means for adult skills for each childhood period. Thus, if in early childhood parents with no schooling would invest the average nutrition inputs parents with high school education do, their children would have 0.0018 SD higher adult skills. The effect in primary school is more considerable than in early childhood, with an 0.0041 SD increase. In later childhood, this effect becomes smaller. The result for schooling differences is the opposite. While closing schooling differences in primary school leads to only marginal improvements, in high school, adult skills increase by 0.0216 SD. If parents with no schooling invest the average of parents with high school education in schooling expenditure in high school, this increases their children's adult skills by 0.0114 SD. Note that this assumes all other factors are fixed. Thus, e.g., adjusting the other inputs might lead to larger effects.

To be able to compare different policies ex-post, one needs to know how changes in cognitive skills translate through child hood. For instance, what happens to adult skills if a policy intervention increases current skills by 0.1 SD? Evans and Yuan (2022) find this effect size to be the average for education interventions. In early childhood, increasing initial skills by 0.1SD leads to an adult skills increase of 0.0004. In primary school, an increase of current skills of 0.1 leads to 0.0041 higher adult skills. In high school, the same effect leads to 0.0216 SD higher skills. However, skills are found to be more amenable early in life. Thus, it might be more complex/expensive in high school to reach the same effect sizes as in early childhood.

In terms of preference parameters parents vary by education (see table 5.3). Parents with higher education value cognitive skills less than their lower-educated peers compared to consumption. This is the case for the utility of current skills. Regarding future skills, parents with high education have a slightly higher valuation. In the last period, the total valuation is $\alpha_e \gamma_e$, both parameters multiplied. Given that, the valuation for skills also in the last period of childhood is higher for parents with no schooling than the ones with high school education. Thus, parents with lower education invest less in their children is not driven by their preferences. The preference for assets, ζ, after the child becomes an adult indicates that parents value assets. This parameter is not allowed to vary by education and, therefore, is the same for all groups.

TABLE 5.3: Calibrated preference parameters

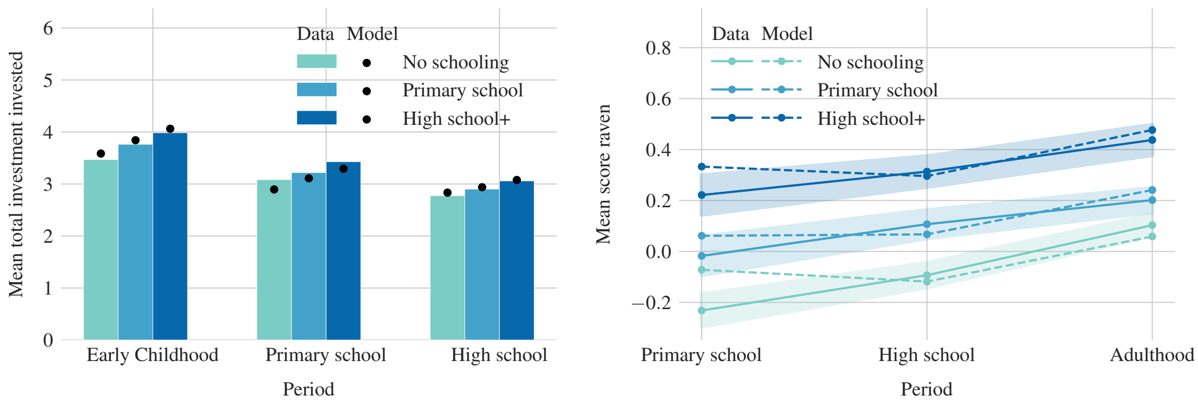
	Parental education:		
	No schooling	Primary school	High school+
For current skills:			
α_e	2.38	1.62	0.98
For final skills:			
γ_e	1.40	1.38	1.45
For final assets:			
ζ	9.98	9.98	9.98

Note: Calibration method used: simulated methods of moments. Moments targeted were investments by parental education and by childhood period.

Regarding their children's skills, if anything, parent's budget constraint or their productivities keep them from investing more in their children. These utility parameters are derived assuming that parents fully know the skill formation process. Dizon-Ross (2019) and Cunha et al. (2020) find that parents with lower education overestimate the impact of their skills and underestimate the persistence of current skills. Thus, they invest less than optimal in this scenario and should invest more. As I do not account for this type of imperfect knowledge in the model, the optimal value is the one observed. Hence, preference parameters are derived for these values indicating the utility derived in contrast to the one from consumption. These parents would invest more without the knowledge barrier, lowering their consumption, and the value for preferences would be even higher. Therefore, the values found here are instead the lower bound of parameters.

Regarding the model fit, I will display first the targeted moments, thus, the moments I match in the simulated methods and moments. Second, I will display untargeted moments, which are not matched in the estimation procedure. Here, I chose the skill formation by parental education group, as these outcome and process is important for policy analysis. Comparing the targeted moments of the model with the data shows that the model does reasonably well (see table A.6). The model fits the data well regarding investments and untargeted moments for skills, as shown in figure 6. If anything, total investments in the early and primary school periods seem slightly off in the model simulations. Regarding the untargeted moments of nutrition and schooling, figure 14 shows the fit. The model fits schooling investment in primary school well and tends to simulate too high levels of schooling expenditure in high school and generally too low nutrition investments in both periods. The gap between parents of different education is fitted well, however. Looking at untargeted moments on raven test scores, I match well the horizontal gap between parents from different education backgrounds. I also fit the gap vertically well between high school and adult skills. In primary school, the levels of skills are slightly off. In figure 13, displaying the result for math test scores, the curvature of the skill gap is better captured, but the level for low-educated parents in primary school is still off. As the model's focus is not on early childhood, I concentrate the analysis on policy experiments in primary and high school.

As these parameters are modeling the skill formation process well, I can now use them to simulate the skill gap by socioeconomic status and for policy experiments. For these, it is vital to keep in mind that they will only use the estimated parameters,



(A) Targeted moments: investments

(B) Untargeted moments: skills

FIGURE 6: Model fit for investment choices and skills by period and parental education
Note: Investment and skill means plotted by parental education and childhood periods.

thus not capturing behavioral responses which are not modeled. Thus, I will not be able to account for differences in, e.g., school quality or network effects, other than the parts captured by parental education productivities or parenting skill types.

Because of data limitations, I also cannot model time investments - the time parents spend actively with their children - well. In general, Del Boca et al. (2014) find parental time to matter most in early childhood and monetary investments in later childhood. Hence, later periods should be less impacted by this modeling choice. My focus is on modeling the whole childhood period, not only early childhood. Thus, more insights on monetary investments can complement the literature by adding results on other investment inputs and the transformation to adult skill outcomes.

Also, I do not observe children in between periods and do not impose assumptions on the inputs in between periods. Therefore, I only model skill development by period and control for the age I observe the child. I abstract from modeling intra-household allocation and investment trade-offs between siblings due to data constraints and complexity, although household poverty might be shared unequally (Calvi (2020)). I account for the number of siblings in the schooling productivity and for the number of children and adult household members in the income estimation. Further, I adjust household consumption with equivalence scales (for details, see appendix A.3). To limit the impact on the results, I control for household size and amount of siblings in the estimation and calibration. Additionally, I only use food diversity as a measure, not quantity, which is more likely to be impacted by disproportionate sharing.

In this context, gender and ethnic group investment differences might also play a role, as Ashraf et al. (2020) find that Indonesian parents who have the tradition of bride prices invest more in a girl's education after an education policy. I control for gender in the investment function estimations but not in overall levels of investments. I did not find significant differences in education expenditure by gender for groups with bride prize traditions in general. The sample size might drive this null result. In my sample, the share of children who grew up in families with a bride price tradition is not high at 17%. In general, Maccini and Yang (2009) find evidence for investment differences by gender in nutrition allocation in times of hardship in Indonesia. However, these findings are in the context of in utero exposure, a period which I do not model. Nonetheless, future work might extend the analysis and model on this notion to lead to more detailed results.

6 Decomposing the skill gap

Using the models, I can quantify how parental socioeconomic background drives the skill gap. To do so, I shut down potential channels one by one in the model and report simulated results in 6.1. For simplicity, I compare parents with high school education and parents without schooling. For the drivers, I will start with differences in preferences. Then I will close technology differences in the skill production function by education. Lastly, I will account for the different levels of income and assets.

TABLE 6.1: Skill gap decomposition

	Investment gap (%)	Adult skill gap (std.)
Baseline gap	10.59	0.35
Closing the gap by:		
Preferences	88.80	0.49
+ Investment productivities	103.29	0.53
+ Skill productivities	103.29	0.20
+ Income	15.77	0.05
+ Assets	-0.31	0.00

Note: Gaps indicated are between high school parents and parents with no schooling. Rest of the gap derives from differences in initial skills and prices and survey year.

Preferences for skills are lower for parents with high school education. When I close this gap, parents with no schooling have the same value for cognitive skills as parents with high school education. Given their smaller budget, they will invest less in their children than they do in the status quo. Therefore the investment gap increases to 88.80%. This increase translates into a skill gap of 0.49 SD. If parents with lower levels of education did not value their children's skills more than parents with high school education do, the gap in cognitive skills would be 0.14 SD larger.

The next step is to close the gap in the productivity of schooling. Parents with high school education have higher productivity of schooling than parents without schooling. This productivity leads them to shift inputs toward schooling expenditures away from nutrition, given the same total investment level. Given this shift, their total price of investments increases. This price increase happens because goods are complementary, and increasing one nutrition unit is cheaper than the same amount in expenditure units. This relation is reflected in the total price of investments, which varies for each parent (see equation 29). Closing these differences leads to higher prices for parents without schooling, resulting in a bigger investment gap. This gap increases the adult skill gap to 0.53 SD. However, the increase is small compared to one of the other drivers. Another difference is the difference in total factor productivity. This productivity describes the ability to transform current skills and investment into future skills. The higher the productivity, the higher future skills for the same level of investments and current skills (see 5 for details). Parents without schooling have lower productivity than parents with high school education. Therefore, assigning them one of the parents with a high school education closes the skill gap to 0.2 SD. It does not change the investment gap, as this productivity does not influence investment levels.

Remaining sources of the socioeconomic skill gap are differences in income and assets. Closing income differences reduces the investment gap to 15.77% and the adult skill gap to 0.05 SD. As this decrease is large, income constraints play a significant role in forming the adult skill gap, which means closing income differences can also have significant effects on future generations. Differences in assets constitute most of the rest of the gap. Leftover differences are marginal and mainly steam from differences in initial skill levels, prices by region of residence, and survey year.

To understand further the dynamics of skill development, I plot in figure 7 changes in the skill gap over childhood periods. In early childhood and high school, income and preference differences contribute more to the gap than in primary school. In contrast, in primary school, differences in productivity are more critical. Herefore, lifting income constraints in early childhood and high school is more effective than in primary school.

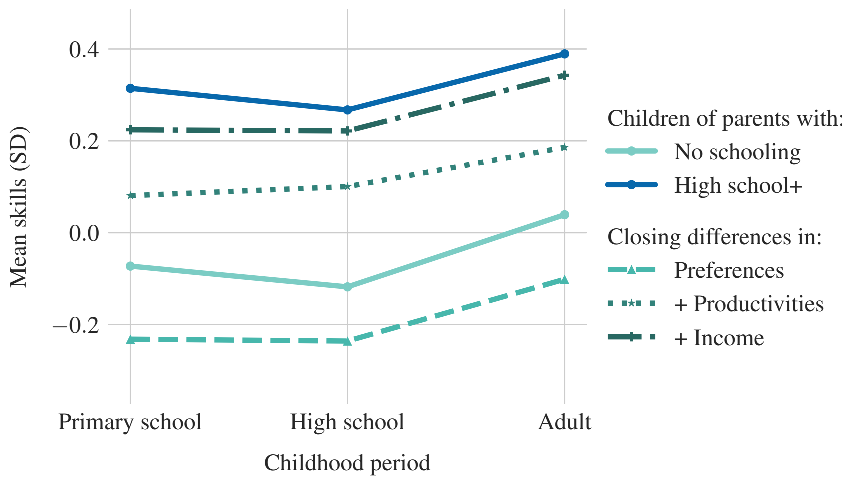


FIGURE 7: Skill gap decomposition

Note: Solid lines represent the existing skill gap between children of parents with no schooling and parents with high school education. Non-solid lines are indicating the skill level of children of parents with no schooling when closing differences in: preferences, productivities and income. To do so, parameters of parents with high school education are assigned to parents with no schooling and skill outcomes simulated.

To compare this with investment gap changes by period, see table A.8. For investment differences, income plays a significant role in all periods but most in the high school period. This significance for the high school period could be driven by the fact that monetary investments become more critical with time, and schooling gets more expensive in high school. Preference differences magnify in high school, same for differences driven by investment productivities, although those are small in comparison.

Income, preferences, and differences in skill production technology are the main drivers for the skill gag. For policies, closing income differences would have significant effects. Targeting the total factor productivity is more challenging. Increasing parents' education would increase productivity and lead to a smaller skill gap. Doing so would also mitigate large parts of the income differences. Due to model constraints, I cannot speak on targeting differences in total factor productivity apart from increasing parental education. Changes in these children's environment might mitigate low productivity. As these are not explicitly modeled here, further extensions of this work are needed to give more clear policy implications.

7 Policy experiments

I simulate three policies, a nutrition price subsidy, a schooling price subsidy, and an unconditional cash transfer. With these policies, I target the children with parents who are in the 20% lowest part of the income distribution. I first simulate the impact of each of these policies on adult skill outcomes. Second, I simulate the impact of combining them. This means for example, allocating the cash transfer and one of the price subsidies. I focus on the last two periods of childhood, thus do not simulate the policies for early childhood as I do not model this period in detail. To ease the comparison of policies, I simulate them to have the same costs.

Given the same costs, the cash transfer has a size of 3% of the mean average income of the lowest 20% of the income distribution. The food price subsidy is around 20%. This subsidy could be implemented using vouchers, which allow parents from the lower part of the income distribution to shop at lower prices. The schooling expenditure subsidy is 99%. This high percentage means that the program pays nearly all the schooling expenditure of the household. One could treat that as a tuition waiver. For costs, I only use the costs I can identify with my simulations. Thus, the monetary amount supplied to households is part of the program's costs but not the implementation costs. This shortcoming needs to be considered for interpretation effects. The lack of implementation costs could be especially relevant for the last two policies, as food price subsidies might impact food prices in general. Further, I do not simulate any other impacts than on the cognitive skills of the targeted children and cannot simulate general equilibrium effects. The simulations' results are summarized in table 7.1.

As one can see, the cash transfer has little impact, supporting the conclusion of limited effects of cash transfers on cognitive skills by Molina Millán et al. (2020) and Baird et al. (2019). A food price subsidy is most effective for the same costs, with an average increase in adult skills of 0.04. A school price subsidy is slightly less effective than a food subsidy, with an increase of 0.03 SD. This result reflects that it is cost-effective to target parental investment behavior via price incentives. By decreasing one input price, both inputs increase in quantity. This behavior is a direct consequence of the complementarity of nutrition and schooling expenditure. The increase in investments is higher than in the case of unconditional cash transfers. Therefore, skill outcomes increase. In general, the input with the price decrease increases more as

Table 7.1: Policy counterfactuals - investment and skill change

	Cash transfer	Nutrition subsidy	Schooling subsidy
Change in mean adult skills (SD):			
All targeted	0.00	0.04	0.03
Change in mean investments (%):			
Investments	1.33	16.20	8.91
Nutrition	1.21	15.82	6.82
Schooling	1.46	18.55	90.38
Costs in 100,000 rupees per child:			
Per 0.01 SD increase	1918.27	210.98	284.31
Total amount	7.64	7.64	7.64

Note: Policies are designed to have the same costs (in 100,000 rupees $\sim$ \$7), resulting in a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy.

optimal shares of inputs change due to different prices. Regarding prior findings in the literature, the increase in food diversity with price subsidies complements findings of Kaul (2018) and Krishnamurthy et al. (2017). These evaluations find a price subsidy in India to increase households food diversity. In contrast Jensen and Miller (2018) do not find any increases in nutrition diversity for a staple subsidy in China. Apart, the evidence on school meals supports my findings. Provision of school meals has been found to increase cognitive skills in several context (see Alderman and Bundy (2012), Frisvold (2015), Chakraborty and Jayaraman (2019) and Aurino et al. (2020)). Additionally, if the healthiness of school meals increases, they yield higher impacts, as found in an intervention in the United Kingdom (Belot and James, 2011). Extending these findings, I further find parents to increase also schooling expenditure, which additionally increases child outcomes.

A detail to note is that total investments into schooling increase little in the schooling subsidy scenario compared to the food subsidy. This behavior is partly driven by period effects. It is most effective for parents to increase investments in high school and less in primary school (see table A.10). In contrast with the food subsidy, parents increase mean investments in both periods. The increase in skills in the high school period translates into adult skills with more persistence than in primary school. Therefore, the schooling subsidy is nearly as effective as nutrition, even if investment levels change less on average. In general, the high degree of complementarity between nutrition and schooling investments leads to strong reactions of parents to price changes.

Table 7.2: Policy combination counterfactuals - investment and skill change

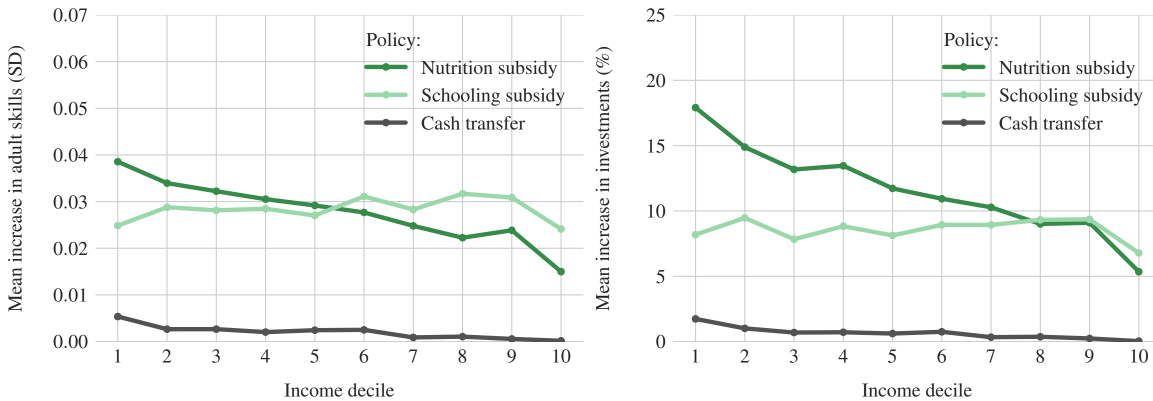
	Cash+ nutrition	Cash+ schooling	Nutrition+ schooling	Nutrition subsidy
Change in mean adult skills (SD):				
All targeted	0.04	0.03	0.07	0.10
Change in mean investments (%):				
Investments	17.59	10.18	26.46	48.52
Nutrition	17.13	8.03	23.90	47.55
Schooling	20.29	93.16	131.73	64.67
Costs in 100,000 rupees per child:				
Per 0.01 SD increase	384.79	494.30	267.91	157.46
Total amount	15.36	15.39	17.42	15.36

Note: Costs are expressed in 100,000 rupees ($\sim$ \$7), combined policies are a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy. The nutrition subsidy is 51% to be cost-equivalent to the cheapest combination.

Combining the policies shows that the interventions have no additional increase in skills when jointly implemented (see table 7.2). Hence there are no significant dynamic complementarities between these two policies when one considers parental responses. However, parents increase their investments, which leads to bigger costs. The increase in skills is effectively lower though, which which is why jointly implemented policies are not cost-effective even if the maximize impact. It is more cost-effective to implement the nutrition subsidy alone.

As these policies are targeted toward the lowest 20% of the income distribution, I now extend the analysis to the entire population to see if there are differential effects. To do so, I simulate the described policies for the full sample, and then plot mean effects by income decile (see figure 8). Overall, I find that nutrition subsidies and cash transfer impacts decrease with income. In contrast, schooling subsidy effects slightly increase. In support of the stronger impact of nutrition subsidies on children from poorer households, Aurino et al. (2020) find poorer children to significantly stronger profit from the proposition of school meals in Ghana.

Nutrition price subsidies incentivize parents in the lower part of the income distribution to invest more in nutrition. In contrast, parents in the upper part of the distribution react to a lesser extent in increasing their investments. The opposite is true for schooling subsidies. Parents in the lower part of the income distribution are



(A) Increase in skills (SD)   (B) Increase in investments (%)

FIGURE 8: Policy impacts by income decile

Note: Plotted are mean increases in cognitive skills and investment changes in percent from baseline by income decile for each policy.

more effective at producing investments with increased nutrition investments and less effective regarding schooling. Consequently, they spend a higher share of investments on nutrition which leads to them reacting stronger to nutrition price changes and a schooling price reduction has smaller effects on children in this part of the income distribution. Additionally, one can observe that unconditional cash transfers mainly increase investments for the lowest part of the income distribution, while later, parents react only marginally in their investments. This pattern indicates that cash transfers can help lift the budget constraint of the ultra-poor. The top parts of the income distribution are not as budget-constrained leading to negligible effects on cognitive skills. Regarding cost-effectiveness, nutrition subsidies still outperform other policies (see table A.11). Given the differential reaction of parents by socioeconomic status, nutrition subsidies reduce inequality in skills most.

Note, that the average increase of investments for schooling are lower than for nutrition in most cases. However, especially for the top part of the income distribution effects are higher. This is driven by the unequal increase of investments by period. The schooling price subsidy mainly increases investments in high school not in primary school. Skills and therefore also earlier investments have a low persistence though, which is why increases in nutrition investments in primary school fade out to some extend until adulthood. Regarding the most disadvantaged, the lowest decile in the income distribution, decreasing costs for nutrition is very effective. Further, for this part of the population, cash transfers have an effect of 0.01 SD on skill development (see table A.11, rounded to the second decimal). This indicates the stringent budget constraint under which these parents operate.

8 Concluding remarks

This paper documents the skill gap for children from different socioeconomic backgrounds in Indonesia. I quantify which drivers contribute to the skill gap in each childhood period: early childhood, primary school, and high school. To do so, I estimate a dynamic structural model of children's skill formation and parental investment decisions on nutrition and schooling. Results show that investments matter, especially in early childhood, and skills become more persistent in later childhood. Nutrition and schooling are complements and more complementary in high school than in primary school.

I explicitly model and quantify drivers of the socioeconomic skill gap among adults and find that parental income and assets contribute to 0.2 SD of the adult skill gap. Mainly, the skill gap is driven by differences in skill production technology by parental education (0.29 SD). These differences are particularly evident in primary school. Importantly, I also find that parental preferences differ across education groups: parents with lower education value their children's skills more than parents with high school education in Indonesia. Thus, the differences in skills are not driven by preferences but mainly by income and skill production productivity. If parents without schooling valued skills like parents with high school education, the skill gap would be 0.14 SD larger than the status quo.

Policies such as nutrition and price subsidies can partly close the skill gap. A nutrition price subsidy targeted to parents in the lowest 20% of the income distribution increases adult skills by 0.04 SD, and a schooling subsidy by 0.03 SD. In contrast, cash transfers have a negligible impact on cognitive skills. If anything, they support the most income-constrained parents investing more in their children. Combining these different policies is not cost-effective. Regarding impacts across the income distribution, the nutrition subsidy increases skills most for the bottom part of the distribution reducing inequality. Similarly, the effects of cash transfers, albeit already small, decline further with income. For the upper part of the income distribution, the effect of subsidizing schooling is higher than the impact of nutrition subsidies. This pattern indicates the stringent budget constraints for the bottom part of the distribution but also that the top part is more effective in utilizing schooling to increase cognitive skills.

Future research could focus on extending the framework used in several dimensions. First, by accounting for information disparities between parents from different socioeconomic statuses and addressing how they influence parents' responses to policies is a potential enrichment of this model. Recent work by Dizon-Ross (2019) and Cunha et al. (2020) shows that parents with lower education are found to overestimate their children's skills and the impact of their investments compared to their peers. They also tend to underestimate the importance of early life investments driven by the persistence of current skills. Closing these information differences could lead to a smaller skill gap. Second, the interplay between time investments and a more detailed modeled first period of childhood and prenatal investment could lead to additional insights into the skill formation process.

Another avenue could be to model intra-household allocation among siblings and the effects older siblings have on the development of cognitive skills of younger ones. Calvi (2020) and Brown et al. (2021) find household poverty to be shared unequally between household members. Knowing if and which children of the household are most impacted by this and in which setting could have implications for the targeting of policies. With richer data on all household members, dynamics might be uncovered. These dynamics could also play a role in the analysis and targeting of policies.

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A Appendix

A.1 Data

A.1.1 Food prices and nutrition investments

To capture nutritional diversity, nutrition investments are proxied by the number of food groups. Food groups in the consumption data are carbons, protein, dairy, vegetables, and fruits. If the household expenditure on one food group is more than 5% of the total expenditure, it is counted as an investment in this food group. Due to data constraints, I cannot identify if household consumption aligns with the child's nutrition. However, I assume that it is a good enough proxy for nutritional diversity since it is unlikely that children receive entirely different food than the one bought by the household. Nutrition diversity is expressed by a measure between 1 and 5, with $n_t = 5$ meaning that a child consumes all five food groups and $n_t = 1$ that it consumes only one food group.

For food prices, I rely on the community surveys in the IFLS, which surveys food prices in the community markets and shops. I construct unit prices of protein, carbons, and vegetables, which are the most prominent consumption expenditure groups and have the most reliable price data (in terms of units).

Then I build the food price by weighting prices by the median consumption fraction for households in the sample consuming all three groups. This leads to a weight of

0.43 for carbs, 0.14 for vegetables, and 0.43 for meat. These prices are then scaled by the average kilograms consumed by households using equivalence scales for Indonesia estimated by Olken (2006) for different ages and household compositions. These are close to the modified OECD scale. I use these equivalence scales and median prices to find the median amount of kg consumed by a household. This amount I then multiply by the factor an additional child of the corresponding age from the household equivalence scale and the median regional food price mentioned above.

A.1.2 Schooling prices and investments

For each household, I have detailed information on what they spend on schooling, e.g., the school fees and books, uniforms, and transport. As investments, I define all registration costs, exam costs, and fees, which the household pays for the child's education. I add the investments into books. I restrain from adding food, uniforms, and transport costs, since I do not assume them to measure the school's quality and influence skill formation. However, this neglects potential budget constraints for these items. The schooling price is assumed to be equal to 1.

A.1.3 Household income and assets

I sum all income reported for the household. This includes business and farm business income, as well as all other income received by any of the household members. Further, this entails non-labor income, the number of transfers, retirement payments, and scholarships received. I adjust household income by the household size for the calibration. For that, I use Olken (2005) equivalence scales derived for Indonesia. As these are derived from aid allocated by the Raskin rice program to different family structures, I assume they will mimic the family's income and how it translates into consumption. Deaton and Zaidi (2002) and Batana et al. (2013) state that the wildly used modified OECD scale or square root scales suit high-income countries. Using the scale for low-income countries might overestimate the degree of the economics of scale, as durables are easier to share than food, a significant fraction of the expenditure in low-income countries. Further, they tend to overestimate the cost of children. Hence, I use Olken's estimated scale, which is higher. Thus the economics of scale are lower.

Most scales are convertible in the following:

$$N^{eq} = (n_a + \alpha n_c)^{\theta} \tag{18}$$

where n_a is the number of adults in the household, and n_c is the number of children. α is the cost of children, and θ expresses the economies of scale. In the scare root scale, $\alpha = 1$ and $\theta = 0.5$. In contrast Olken estimates $\alpha = 0.93$ and $\theta = 0.85$, which confirms Deaton and Zaidi (2002)'s claim that the economies of scale are lower, thus θ higher in low-income countries. This also goes with Santaeulàlia-Llopis and Zheng (2017), who estimate scale parameters in Malawi to be higher than the OECD ones.

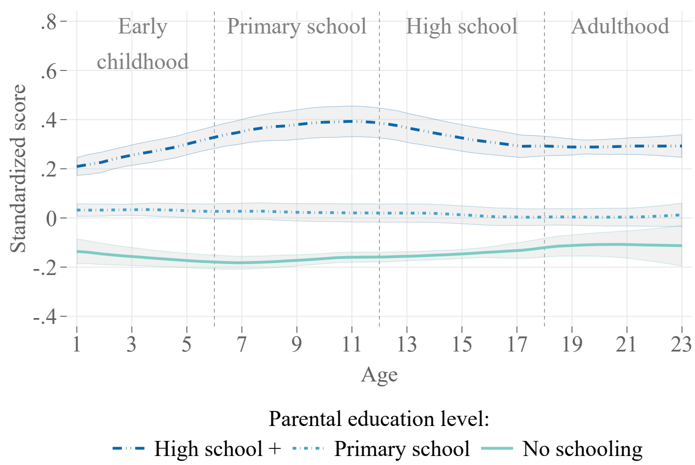
For assets, I sum all assets reported in the data, which are expressed in monetary value. This entails real estate owned, land, livestock, machinery, household appliances, savings, jewellery and furniture. I substract from assets the reported amount of dept of the households. Then I adjust the left-over assets with the household equivalence scale.

A.1.4 Skill measures

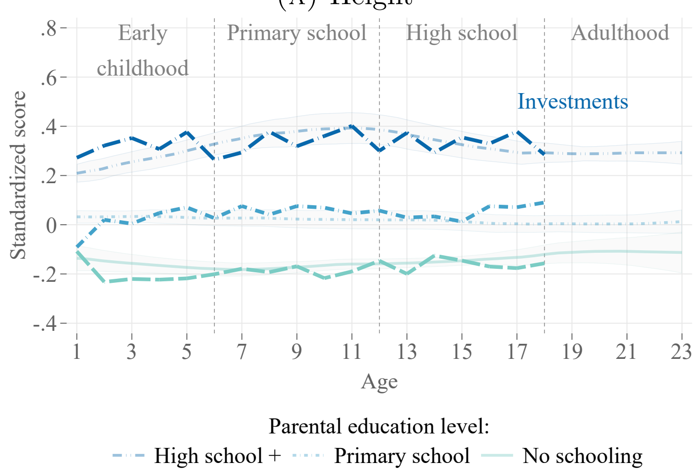
For health skills, the following measures are used: height and weight. With the help of the WHO Child Growth Standards and WHO Reference 2007 composite data files as the reference data, I build z-scores for children under 20 years old (Vidmar et al., 2013). Hereby the height-for-age, weight-for-age, BMI-for-age and weight-for-height z-scores are computed. BMI is taken as an indicator for older individuals, thus the parents and adults. In period 1, early childhood, the measures used are height-for-age and weight-for-age since no cognitive measures are available.

For cognitive skills outcomes, cognitive tests conducted by the survey team are available, which I standardize by age. The IFLS has several test score metrics available: In 1997, a math test with 40 questions was conducted for the following age groups: 7-9, 10-12, and 13-24, and the same was done for a language evaluation. For younger ages, no test scores are available. Therefore, in the early childhood period, only health outcomes can serve as a measure of skills. For 2000, 2007, and 2014 a raven test was conducted with 12 questions, followed by a math test of 5. These were designed in 2 versions, one for age group 7 to 14, the other 15 to 24. In both cases, the number of correct answers is standardized by age and year. Adult respondents answered a cognitive test in 2007 and 2014. The tests ask them to remember ten words for a short period, and a second round asks how many they remember after some minutes. In 2014 additionally, a simple subtraction exercise was asked. Adult test scores are standardized by year to avoid some candidates being counted double. As cognitive measures during childhood, raven or language and math scores are taken, while for adults, an average for word- and math tests is taken.

A.2 Stylized facts and descriptives



(A) Height



(B) Height and investments

FIGURE 9: Children skills and investments over age by parental education

Note: Skills are fitted with local mean smoothing by age and parental education groups. Parental education groups correspond to the average education of both parents. Confidence intervals displayed are at 95% level. Investments plotted are standardized nutrition investments. Scores of skills and investments are standardized by age to have a mean of 0 and SD of 1.

Table A.1: Sample characteristics

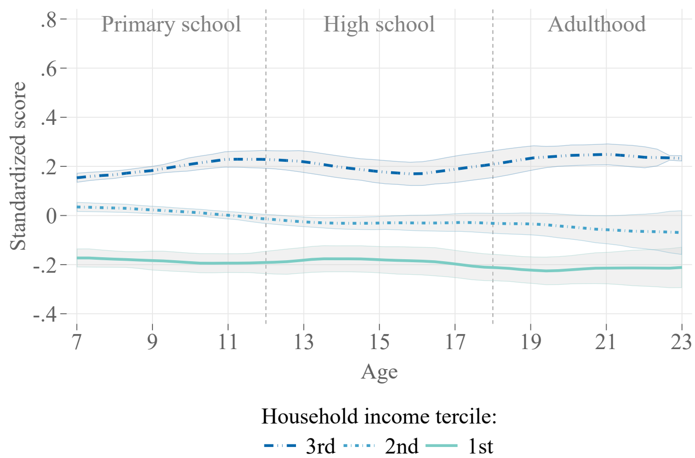
	Mean	SD	Min	Max
Female	0.50	0.50	0	1
Rural	0.54	0.50	0	1
Islam	0.88	0.33	0	1
Mother's years of education	5.50	4.12	0	18
Father's years of education	6.58	4.38	0	18.5
Birth year	1990.88	6.53	1979	2007
Household income	270.65	331.2	0	3982.9
Weight-by-age	-1.16	1.44	-4.99	4.92
Height-by-age	-1.49	1.27	-4.98	4.97
Stunting	0.34	0.47	0	1
Wasting	0.09	0.28	0	1
Mother's age	41.30	9.15	17	78
Father's age	46.84	10.5	20	96
Adult household members	3.93	1.82	0	8
Household members <18	1.86	1.36	0	5
N	19,343			

Note: Monetary values are deflated and reported in 100,000 Rupees.

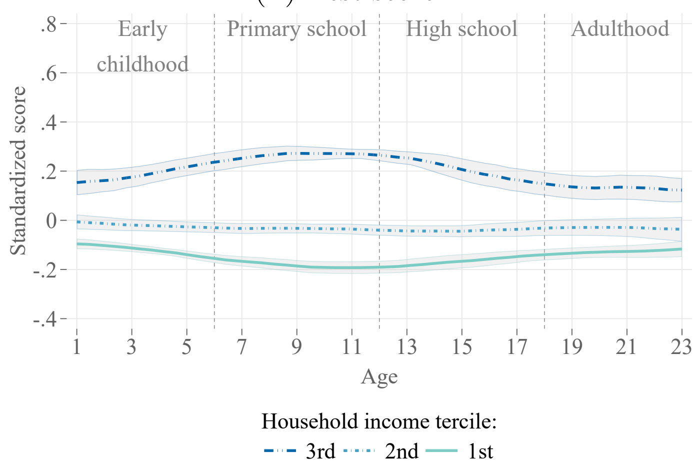
Table A.2: Sample characteristics by period

	Early childhood	Primary school	High school
Food groups	3.67	3.61	3.58
Schooling spending	0.24	2.61	6.00
Age	3.02	8.84	15.34
In school	0.06	0.93	0.73
Observations	4,563	6,329	8,451

Note: Monetary values are deflated and reported in 100,000 Rupees.



(A) Test score



(B) Height

FIGURE 10: Children skills and investments over age by parental income

Note: Corresponding skills are fitted with local mean smoothing by age and household income tercile. Confidence intervals displayed are at 95% level.

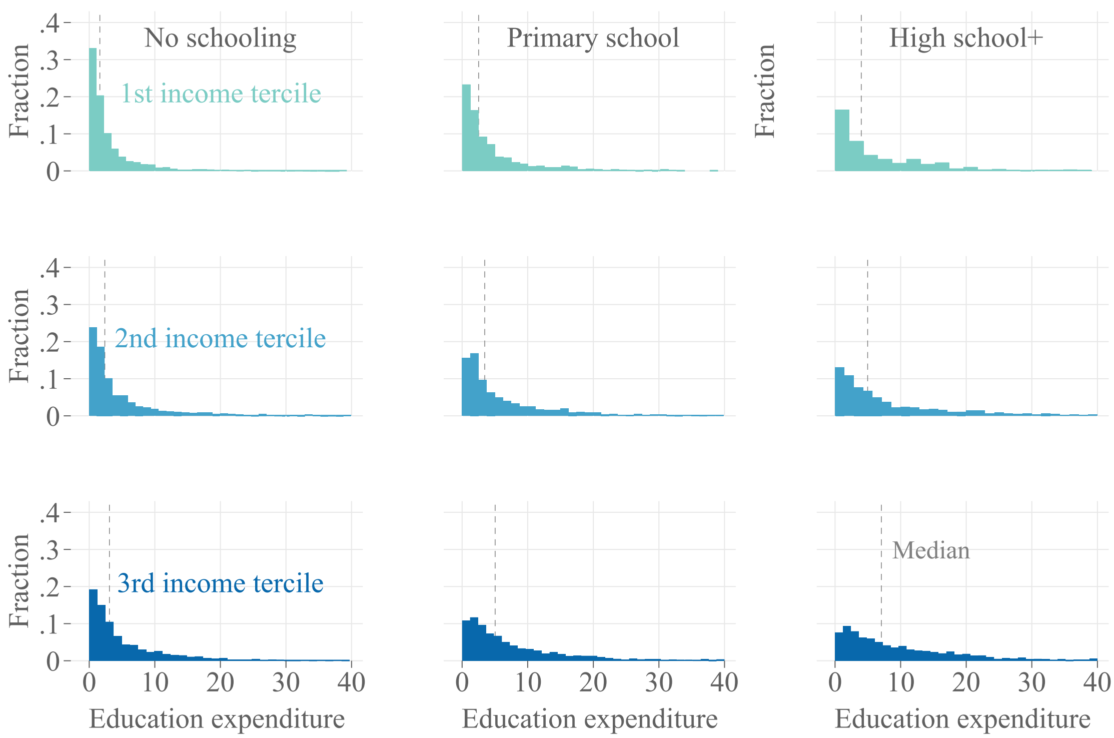


FIGURE 11: Heterogeneity in education spending by parental background

Note: Education spending histograms by parental education level and household income (in terciles). Parental education groups correspond to the average education of both parents. Expenditures are expressed in 100,000 rupees. The grey-dashed line indicates the median value for that category.

A.3 Estimation and calibration details

A.3.1 K-means algorithm

I follow Bonhomme et al. (2022) to estimate the unobserved types of parenting skills outside of the model. To do so, I build means over the life-cycle of schooling, nutrition investments, and household income for each parent couple. I then standardize these and run the k-means clustering procedure, which will allocate each household to the cluster whose moments have the least distance to the cluster mean.

To estimate heterogeneity groups using the k-means clustering algorithm, I need to choose the number of clustering groups K. As this is a data-driven approach, they are not known as apriori, but data can be used to determine it. To do so, I use the commonly used Elbow statistic. For a given number of clusters K, the algorithm minimizes the total within-cluster variance:

$$\min_{k \in \{1,..,K\}^N} \sum_{t=1}^{N} \sum_{c=1}^{C} ||\boldsymbol{m_{t,c}} - \boldsymbol{\overline{m}_k}||^2 = SSE_k \tag{19}$$

To compare Elbow statistics, the variance SSE_k is calculated for each number of clusters run, $k = 1; ...; K_{max}$. These statistics are then plotted against their corresponding number of clusters, as seen in figure 12a. With an increasing number of clusters, the variance decreases as observations within a cluster become more similar. The optimal number of clusters is at the kink in the plot, i.e. the point where the decrease in SSE changes the most. Adding more clusters than at this king would have limited value in explaining the variation in the data. The silhouette criterion in figure 12b. The higher the criteria value, the more the two clusters are from each other. Thus, the borders between them are well defined.

As shown in figure 12a, the elbow criteria determines the optimal amount of clusters K to be 4. The silhouette criterion is maximized at two but also high at 4. To check if the number of clusters drives the results, I run the GMM estimation for $K \in \{2, 3, 4, 5\}$ clusters. As one can see the results for $K = 2$ in table A.12, $K = 3$ in table A.13, $K = 5$ in table A.14 are comparable to the main results in table A.5 with $K = 4$. Coefficients and standard errors only vary marginally. Thus, the amount of clusters does not drive the results and, if anything, adds explanatory power. More clusters seem to explain more unobserved heterogeneity in investments, as schooling productivity varies by type. However, after $K = 4$, the amount of observations decreases by type, as shown in table A.7. Hence, increasing the computational burden further has little reward. This is confirmed by the fact that these amounts exceed the amount determined to be optimal by the elbow criterion.

A.3.2 Household income

To estimate household income, I regress parental education, number of household members (adults and children), rurality and age of the household head, and parenting skills on household income. Additionally, I include year and province fixed effects. Thus:

$$\ln(y_t) = Z'_{y,t}\gamma_y + \eta'\gamma_\eta + \epsilon_{y,t} \tag{20}$$

Here, $Z_{y,t}$ are the named household characteristics that can vary by period. η is the unobserved parenting skills I assume the household income, as it is likely that characteristics resulting in productive parents also translate at least partly into higher wages. Results can be found in table A.4.

I use the resulting coefficients to predict future household income for the calibrations and simulations. Further, I assume the income shocks to be i.i.d. normally distributed. Thus $\epsilon \overset{i.i.d}{\sim} N(0, \sigma_y)$.

A.3.3 Transition of other household characteristics

I assume all household characteristics to be stable over time, except the year, age, and age of the household head. Since period one observations I use for the calibration start are either in 1997 or 2000 for the transition to the next period, I get either 2000 or 2007 for 1997 or 2007 for 2000 (observed for the first period, as I know next period). Afterward, due to the survey design, all future waves are seven years apart. Thus I apply that to simulate the year in which the child is observed in the next period. Then apply this gap to its age and the father's age.

Knowing the next year then allows me to allocate the correct food price for the given community in that year to the simulated period. Thus, I assume they do not move. For now, I assume the number of household members and other children in the household to be stable across childhood, the same for the location in a rural or urban area. To relax this assumption could be a potential future extension.

A.3.4 Skill formation estimation

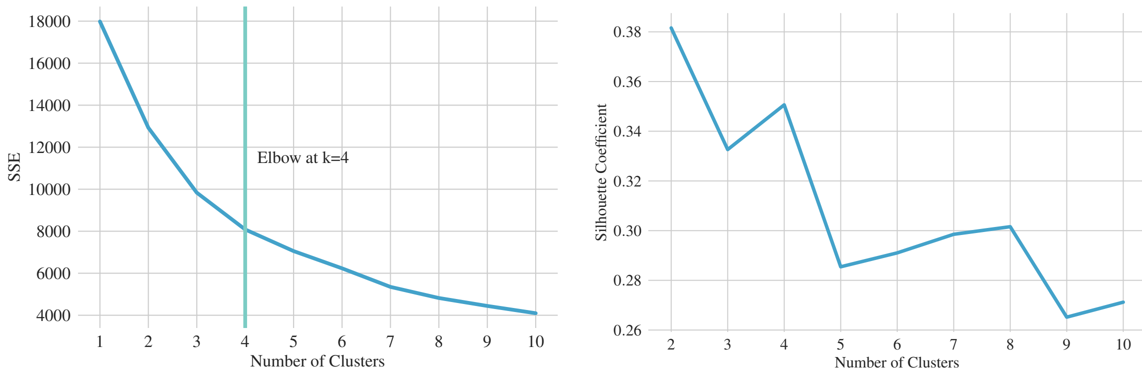
Regarding the GMM estimation, two obstacles driven by data constraints occur. Firstly, only nutrition inputs are available to measure investments in the first period. Thus, there is no stage with relative investment input ratios, which can then be plugged into the human capital parameters. Hence the food groups are directly plugged into this equation. Further, I do not observe cognitive skills in the early childhood. Hence, I use height and weight as a proxy. Therefore, $\delta_{2,1}$, the persistence of skills cannot be directly compared to the parameters in later periods, as it measures the persistence of height and weight on future cognitive skills.

Second, I assume nutrition is unconstrained, however I only observe food groups up to five. Therefore, I conduct robustness checks in case it is constrained to 5. If nutrition is constrained, the optimal demand ratios for the GMM moments hold only if $n_t < 5$ (see A.6 for details for $n_t = 5$). In the main specifications, I also include $n_t = 5$, assuming that it does not drive the results. As a robustness check, I dropped them and ran the results without using observations with $n_t = 5$ to estimate the relative demand equations (see table A.15). The results are relatively similar, which indicates that this subgroup does not drive the general results. If anything, the estimates are less precise, but this could also come from the smaller sample. However, dropping them introduces selection. Thus the results have to be taken with a grain of salt. Future work should exploit how these constraints bias the estimation results. For the calibration, I calibrate the model with and without the constraint without assets and do not see substantial differences. As with assets the constraint induces complex solutions, I then proceed without constraint, assuming that I observe only up to 5 food groups which can translate into 5 or more as investment in reality.

Calibration

To calibrate the model, I use the optimal solution for investments and assets in equations 53 and in section A.6. I match model and data investment means by parental education and childhood period and assets by childhood period to get γ_e and α_e and ζ. To calibrate the model, I use the data from period one and simulate periods two to four with it, to then compare it to the data I observe in those periods in the survey. For a_{min}, the maximum amount households can borrow, I use the average debt I observe in the data in a given year.

A.4 Estimation results



(A) Elbow criterion (B) Silhouette criterion

FIGURE 12: Criterion plots to determine number of clusters for parenting skills

Note: K-means algorithm run for different number of clusters to determine correct number for the following estimation. Plotted are on the right-hand side the within cluster variance, on the left-hand side the Silhouette coefficient by number of clusters used.

TABLE A.3: Characteristics of parenting skill types η

	Averages for type:			
Variable	0	1	2	3
Food investments	0.73	-0.67	0.43	0.58
Education investment	-0.05	-0.30	2.72	0.51
Household income	-0.06	-0.35	0.48	2.97
Fraction mothers with primary school	0.27	0.24	0.14	0.20
Fraction mothers with high school	0.30	0.14	0.55	0.55
Observations	2,613	2,774	356	252

Note: This table displays summary statistics for each of the four clustering groups resulting from k-means procedure. All variables are life-cycle averages and standardized to have mean 0 and standard deviation 1.

Table A.4: Estimation results for household income

	Log(income)	
Father primary education	0.152***	(0.014)
Father high school+	0.422***	(0.016)
Mother primary education	0.112***	(0.014)
Mother high school+	0.294***	(0.017)
Parenting type 1	-0.375***	(0.012)
Parenting type 2	0.296***	(0.027)
Parenting type 3	1.066***	(0.026)
Father age	0.053***	(0.003)
Father age squared	-0.001***	(0.000)
Rural area	-0.348***	(0.012)
Adult household members	0.104***	(0.003)
Non-adult household members	0.016***	(0.004)
Constant	3.109***	(0.079)
Year fixed effects	Yes	
Province fixed effects	Yes	
Observations	36,169	

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1.

Table A.5: Estimation results for skill formation parameters

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.75	(0.86)***	-11.38	(5.11)**
Implied elasticity			0.21		0.08	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.68	(0.51)***	-42.17	(16.55)**
Mother primary			1.10	(0.25)***	3.06	(1.32)**
Mother high			1.87	(0.39)***	5.04	(2.15)**
Father primary			0.09	(0.16)	0.63	(0.47)
Father high			-0.08	(0.19)	0.51	(0.50)
Age			-0.05	(0.04)	3.14	(1.30)**
Female			0.05	(0.13)	1.29	(0.61)**
Rural area			-2.64	(0.53)***	-5.19	(2.22)**
No. of siblings			-0.73	(0.14)***	-2.14	(0.88)**
Mother not Islam			0.39	(0.22)*	1.68	(0.85)**
Parenting type 1			-0.24	(0.14)*	0.06	(0.34)
Parenting type 2			4.74	(0.97)***	9.62	(4.10)**
Parenting type 3			1.64	(0.50)***	2.47	(1.29)*
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.18	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.22	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.11	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)**	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	1.00	(0.07)	1.07	(0.01)	1.09	(0.01)
$\lambda_{4,ts}$					1.21	(0.04)
Observations	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

A.5 Calibration results

Table A.6: Model fit - targeted moments

	Model	Data	SD	Difference
No schooling:				
Early childhood	3.59	3.47	0.80	0.14
Primary school	2.90	3.08	0.94	-0.20
High school	2.83	2.78	1.13	0.05
Primary school:				
Early childhood	3.84	3.76	0.83	0.10
Primary school	3.11	3.22	0.99	-0.11
High school	2.94	2.90	1.16	0.03
High school+:				
Early childhood	4.06	3.98	0.80	0.10
Primary school	3.29	3.43	1.08	-0.12
High school	3.08	3.06	1.26	0.01
Assets:				
Early childhood	620.36	763.38	829.21	-0.17
Primary school	819.23	937.98	1045.17	-0.11
High school	1222.85	1128.23	1172.96	0.08

Note: Calibration method used: simulated methods of moments. Differences are expressed in standard deviations. Values are total investments by parental education and childhood period and for assets by period.

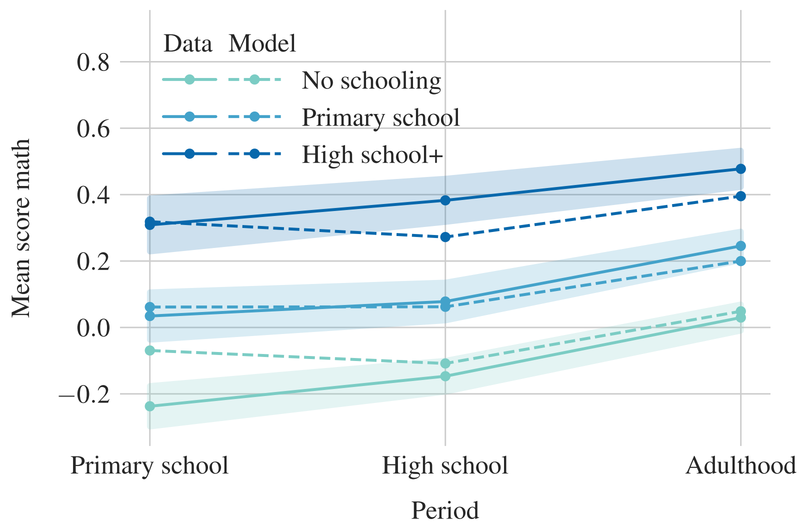
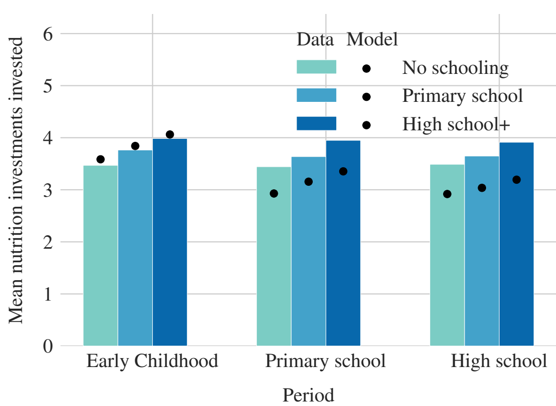
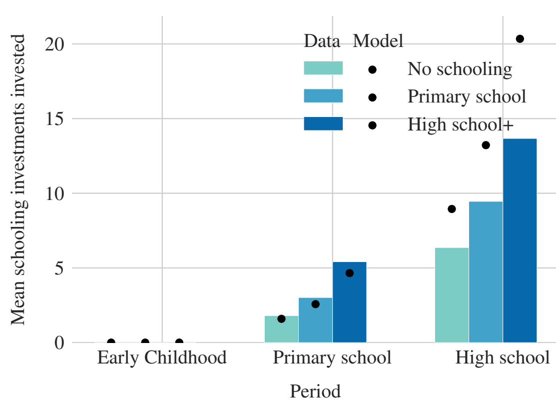


FIGURE 13: Model fit for untargeted children's skills by period



(A) Nutrition



(B) Schooling expenditure

FIGURE 14: Untargeted moments for investment input choices by period

Note: Investment inputs means plotted by parental education and childhood periods. Black dots are corresponding simulated moments.

A.6 Derivation Formulas:

Inter-temporal solution n_t and s_t and relative demands

To derive the relative demands we take first-order conditions for the minimization problem:

$$\begin{aligned}&\min_{n_t,s_t} \quad p_{n,t}n_t + p_{s,t}s_t \\ &\text{s.t. } It = [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}\end{aligned} \tag{21}$$

The Lagrangian looks the following:

$$\mathcal{L} = p_{n,t}n_t + p_{s,t}s_t - \lambda_{1,t}(I_t - [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}) \tag{22}$$

Deriving first order conditions in period 2 and 3:

$$\frac{\partial \mathcal{L}}{\partial s_t} = p_{s,t} - \lambda_{1,t}(a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t-1}\rho_t) \times [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}-1}\frac{1}{\rho_t} = 0 \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = p_{n,t} - \lambda_{1,t}(n_t^{\rho_t-1}\rho_t) \times [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}-1}\frac{1}{\rho_t} = 0 \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{1,t}} = I_t - [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} = 0 \tag{25}$$

Taking ratios $\frac{\frac{\partial \mathcal{L}}{\partial n_t}}{\frac{\partial \mathcal{L}}{\partial s_t}}$ leads:

$$\frac{p_{n,t}}{p_{s,t}} = \frac{n_t^{\rho_t-1}}{a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t-1}} \tag{26}$$

which allows to get n_t in terms of s_t:

$$n_t = \left(\frac{p_{n,t}}{p_{s,t}}a_{s,t}(Z_{s,t},\eta)\right)^{\frac{1}{\rho_t-1}} s_t = \Phi_1 s_t \tag{27}$$

and vice versa:

$$s_t = \Phi_1^{-1} n_t \tag{28}$$

Price for total investments Λ_t and relative demands I_t and I_{t+1}

The price for total investments I_t is supposed to mimic the cost for one unit of investment, thus:

$$\begin{aligned} E_t &= \Lambda_t I_t \\ \Lambda_t &= \frac{E_t}{I_t} \\ \Lambda_t &= \frac{p_{n,t} n_t + p_{s,t} s_t}{[a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}} \end{aligned} \tag{29}$$

To calculate prices we use 27 to get expressions for n_t in terms of s_t:

$$n_t = \left(\frac{p_{n,t}}{p_{s,t}} a_{s,t}(Z_{s,t}, \eta) \right)^{\frac{1}{\rho_t - 1}} s_t = \Phi_1 s_t \tag{30}$$

Replacing n_t in yields in 29 with moving s_t out of E_t:

$$\begin{aligned} \Lambda_t &= \frac{s_t(p_{s,t} + p_{n,t}\Phi_1)}{[a_{s,t}(Z_{s,t}, \eta) s_t^{\rho_t} + (\Phi_1 s_t)^{\rho_t}]^{\frac{1}{\rho_t}}} \\ &= \frac{(p_{s,t} + p_{n,t}\Phi_1)}{[a_{s,t}(Z_{s,t}, \eta) + \Phi_1^{\rho_t}]^{\frac{1}{\rho_t}}} \end{aligned} \tag{31}$$

Intra-temporal solution for I_t

We can use the total price of investment equation 29 for the maximization problem to derive solutions for I_t, c_t and a_{t+1} :

$$\begin{aligned} V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, I_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\ & + \beta_t V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ \text{s.t.} \quad c_t + \Lambda_t I_t + a_{t+1} &= (1+r)a_t + y_t \\ a_{t+1} &\geq a_{min,t} \\ \text{with} \quad \Psi_{t+1} &= \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \\ V_{T+1}(\Psi_{T+1}) &= \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \\ u(c_t) &= \ln(c_t) \\ v(\Psi_t) &= \ln(\Psi_t) \end{aligned} \tag{32}$$

Which gives the Lagrangian:

$$\mathcal{L} = u(c_t) + \alpha_e v(\Psi_t) + \beta_t V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\ - \lambda_t(c_t + \Lambda_t I_t + a_{t+1} - (1+r)a_t - y_t) - \xi_t(a_{min,t} - a_{t+1}) \quad (33)$$

T=3 here, because the period 3 is the last one, where the household makes decisions. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial I_t} = \beta_t \frac{\partial V_{t+1}}{\partial I_t} - \lambda_t \Lambda_t = 0 \quad (34)$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = u'(c_t) - \lambda_t = 0 \quad (35)$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -\lambda_t + \xi_t + \mathbb{1}\{t < T\}(\lambda_{t+1}\beta_{t+1}(1+r)) + \mathbb{1}\{t = T\}\beta_t \frac{\partial V_{T+1}}{\partial a_{T+1}} \quad (36)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = c_t + \Lambda_t I_t + a_{t+1} - (1+r)a_t - y_t = 0 \quad (37)$$

$$\frac{\partial \mathcal{L}}{\partial \xi_t} = a_{min,t} - a_{t+1} = 0 \quad (38)$$

$$(39)$$

Following these one can derive a solution for I_t. First one needs to derive after I_t, which will vary by period due to the continuation value. In period 3, the continuation value looks the following:

$$\begin{aligned} \beta_t V_{T+1}(\Psi_{T+1}) =& \beta_t(\alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1})) \\ \text{with } \Psi_{t+1} =& \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \end{aligned} \quad (40)$$

Plugging it in V_{t+1}:

$$\beta_t V_{t+1}(\Psi_{t+1}) = \beta_t(\alpha_e \gamma_e \ln(\theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}}) + \zeta \ln(a_{T+1})) \quad (41)$$

Thus:

$$\beta_t \frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta_t \delta_{1,t} \alpha_e \gamma_e}{I_t} = \frac{K_t}{I_t} \quad (42)$$

For period 2:

$$\beta_t V_{t+1}(\Psi_{t+1}) = \beta_t(u(c_{t+1}) + \alpha_e v(\Psi_{t+1})) + \beta_{t+1}\beta_t(\alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1})) \quad (43)$$

which is:

$$\begin{aligned}\beta_t V_{t+1}(\Psi_{t+1}) =& \beta_t(\ln(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t}))I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}}) \\ &+ \beta_{t+1}\beta_t(\alpha_e\gamma_e \ln(\theta_{t+1}(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}\Psi_{t+1}^{\delta_{2,t+1}}) + \zeta\ln(a_{t+2}))\end{aligned} \tag{44}$$

plugging in Ψ_{t+1}:

$$\begin{aligned}\beta_t V_{t+1}(\Psi_{t+1}) =& \beta_t(\ln(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})) \\ &+ \beta_{t+1}\beta_t(\alpha_e\gamma_e \ln(\theta_{t+1}(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}}) \\ &+ \zeta\ln(a_{t+2}))\end{aligned} \tag{45}$$

Thus:

$$\beta_t\frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta_t\delta_{1,t}(\alpha_e + \beta_{t+1}\delta_{2,t+1}\gamma_e\alpha_e)}{I_t} = \frac{K_t}{I_t} \tag{46}$$

For period 1:

$$\begin{aligned}\beta V_{t+1}(\Psi_{t+1}) = \beta_t(u(c_{t+1}) + \alpha_e v(\Psi_{t+1})) + \beta_{t+1}\beta_t(u(c_{t+2}) + \alpha_e v(\Psi_{t+2})) \\ + \beta_{t+2}\beta_{t+1}\beta_t(\alpha_e\gamma_e\ln(\Psi_{t+3}) + \zeta\ln(a_{t+3}))\end{aligned} \tag{47}$$

Resulting in:

$$\begin{aligned}\beta_t V_{t+1}(\Psi_{t+1}) =& \beta_t(u(c_{t+1}) + \alpha_e \ln(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})) + \beta_{t+1}\beta_t(u(c_{t+2}) \\ &+ \alpha_e\ln(\theta_{t+1}(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}(\theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}})) \\ &+ \beta_{t+2}\beta_{t+1}\beta_t(\alpha_e\gamma_e\ln(Z_{\theta,t+2})I_{t+2}^{\delta_{1,t+2}}(\theta_t(Z_{\theta,t+1})I_{t+1}^{\delta_{1,t+1}}(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}})^{\delta_{2,t+1}})^{\delta_{2,t+2}}) \\ &+ \zeta\ln(a_{t+3}))\end{aligned} \tag{48}$$

Giving:

$$\beta\frac{\partial V_{t+1}}{\partial I_t} = \frac{\beta_t\delta_{1,t}(\alpha_e + \beta_{t+1}\delta_{2,t+1}(\alpha_e + \beta_{t+2}\delta_{2,t+2}\gamma_e\alpha_e))}{I_t} = \frac{K_t}{I_t} \tag{49}$$

Using the FOCs for c_t and I_t, and the values above for K_t, results in:

$$\frac{\partial\mathcal{L}}{\partial I_t} = \frac{K_t}{I_t} - u'(c,t)\Lambda_t = 0 \tag{50}$$

Now to derive an optimal solution for I_t, I use:

$$c_t = -\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t \tag{51}$$

plugging in:

$$\begin{aligned}
\frac{K_t}{I_t} - \frac{\Lambda_t}{-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t} &= 0 \\
\frac{\Lambda_t}{-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t} &= \frac{K_t}{I_t} \\
(-\Lambda_t I_t - a_{t+1} + (1+r)a_t + y_t)K_t &= \Lambda_t I_t \\
(-a_{t+1} + (1+r)a_t + y_t)K_t &= \Lambda_t I_t + K_t \Lambda_t I_t
\end{aligned} \tag{52}$$

Thus, the optimal solution for I_t:

$$I_t = \frac{K_t(-a_{t+1} + (1+r)a_t + y_t)}{\Lambda_t(1+K_t)} \tag{53}$$

This solution can also be used for period 1, as $I_t = n_t$ and $\Lambda_t = p_{n,t}$. For the borrowing constrained case, $a_{t+1} = a_{min,t}$, for the non-borrowing constrained case, an optimal solution for a_{t+1} is needed, which is derived in section A.6. If $a_t = 0$ and there are no assets, the amount of I_t depends apart from the parameters and related characteristics only on household income y_t.

Optimal solution for s_t and n_t

With I_t one can derive n_t and s_t:

$$I_t = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + (\Phi_1 s_t)^{\rho_t}]^{\frac{1}{\rho_t}} = [a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}} s_t \tag{54}$$

using equation 53 for I_t:

$$\frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)} = [a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}} s_t \tag{55}$$

$$s_t = \frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)[a_{s,t}(Z_{s,t}, \eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{56}$$

With equation 27:

$$n_t = \Phi_1 \frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)[a_{s,t}(Z_{s,t},\eta) + (\Phi_1)^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{57}$$

Optimal solution for a_{t+1} and n_t

From the FOC of the optimization problem, one can use:

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -\lambda_t + \xi_t + \mathbb{1}\{t < T\}(\lambda_{t+1}\beta_t(1+r_{t+1})) + \mathbb{1}\{t = T\}\beta_t \frac{\partial V_{T+1}}{\partial a_{T+1}} \tag{58}$$

If the household is not borrowing constraint: $\xi_t = 0$. For period 3:
Equation 58 results in:

$$\frac{1}{-\Lambda_t I_t - a_{t+1} + (1+r_t)a_t + y_t} = \beta_t \zeta \frac{1}{a_{t+1}} \tag{59}$$

Plugging in the optimal solution for I_t in equation 53:

$$\begin{aligned} \beta_t\zeta(-\frac{K_t(-a_{t+1} + (1+r_t)a_t + y_t)}{(1+K_t)} - a_{t+1} + (1+r_t)a_t + y_t)) &= a_{t+1} \\ \frac{\beta_t\zeta}{K_t+1}(-a_{t+1} + (1+r_t)a_t + y_t) &= a_{t+1} \\ a_{t+1} + \frac{\beta_t\zeta}{K_t+1}a_{t+1} &= \frac{\beta\zeta}{K_t+1}((1+r_t)a_t + y_t) \end{aligned}$$

Follows:

$$a_{t+1} = \frac{\beta_t\zeta}{(1+\beta_t\zeta+K_t)}((1+r_t)a_t + y_t) \tag{60}$$

And for I_t:

$$I_t = \frac{K_t(-(\frac{\beta_t\zeta}{(1+\beta_t\zeta+K_t)}((1+r_t)a_t + y_t)) + (1+r_t)a_t + y_t)}{\Lambda_t(1+K_t)} \tag{61}$$

Which leads to:

$$I_t = \frac{K_t}{\Lambda_t(1+K_t+\zeta\beta_t)}((1+r_t)a_t + y_t) \tag{62}$$

For period 2:

$$\lambda_t = \lambda_{t+1}\beta_t(1 + r_{t+1})$$

$$-\Lambda_{t+1}I_{t+1} - a_{t+2} + (1 + r_{t+1})a_{t+1} + y_{t+1} = \beta_t(1 + r_{t+1})(-\Lambda_t I_t - a_{t+1} + (1 + r_t)a_t + y_t)$$

$$\begin{aligned} &-(\frac{K_{t+1}((1 + r_{t+1})a_{t+1} + y_{t+1})}{(1 + K_{t+1} + \beta_{t+1}\zeta)}) - a_{t+2} + (1 + r_{t+1})a_{t+1} + y_{t+1} = \\ &\qquad \beta_t(1 + r_{t+1})(-(\frac{K_t(-a_{t+1} + (1 + r)a_t + y_t)}{(1 + K_t)}) - a_{t+1} + (1 + r_t)a_t + y_t) \end{aligned} \tag{63}$$

Plugging in a_{t+2} and $A = (1 + \beta_{t+1}\zeta + K_{t+1})$:

$$\begin{aligned} &-(\frac{K_{t+1}}{A}((1 + r_{t+1})a_{t+1} + y_{t+1}) + \frac{1 + K_{t+1}}{A}(1 + r_{t+1})a_{t+1} + y_{t+1}) = \\ &\qquad \beta_t(1 + r_{t+1})\frac{1}{(1 + K_t)}(-a_{t+1} + (1 + r_t)a_t + y_t) \end{aligned} \tag{64}$$

$$\begin{aligned} &\frac{1}{A}((1 + r_{t+1})a_{t+1} + y_{t+1}) = \\ &\qquad \beta_t(1 + r_{t+1})\frac{1}{(1 + K_t)}(-a_{t+1} + (1 + r_t)a_t + y_t) \end{aligned} \tag{65}$$

$$\begin{aligned} \frac{1}{A}(a_{t+1} + \frac{y_{t+1}}{(1 + r_{t+1})}) &= \beta_t\frac{1}{(1 + K_t)}(-a_{t+1} + (1 + r_t)a_t + y_t) \\ \frac{1}{A}(a_{t+1}) + \frac{\beta_t}{1 + K_t}a_{t+1} &= -\frac{1}{A}\frac{y_{t+1}}{(1 + r_{t+1})} + \frac{\beta_t}{1 + K_t}((1 + r_t)a_t + y_t)) \end{aligned}$$

Follows:

$$a_{t+1} = \frac{\beta_t A}{1 + K_t + \beta_t A}((1 + r_t)a_t + y_t) - \frac{1 + K_t}{1 + K_t + \beta_t A}\frac{y_{t+1}}{(1 + r_{t+1})} \tag{66}$$

Plugging in optimal solutions leads to:

$$I_t = \frac{K_t}{\Lambda_t(1 + K_t + \beta_t A)}((1 + r_t)a_t + y_t + \frac{y_{t+1}}{(1 + r_{t+1})}) \tag{67}$$

For period 1, following a similar strategy as in period 2, this yields, with $B = (1 + K_{t+1} + \beta_{t+1}(1 + \beta_{t+2}\zeta + K_{t+2}))$:

$$a_{t+1} = \frac{\beta_t B}{1 + K_t + \beta_t B}((1 + r_t)a_t + y_t) - \frac{1 + K_t}{1 + K_t + \beta_t B}(\frac{y_{t+1}}{(1 + r_{t+1})} + \frac{y_{t+1}}{(1 + r_{t+1})(1 + r_{t+2})}) \quad (68)$$

$$I_t = \frac{K_t}{\Lambda_t(1 + K_t + \beta_t B)}((1 + r_t)a_t + y_t + \frac{y_{t+1}}{(1 + r_{t+1})} + \frac{y_{t+1}}{(1 + r_{t+1})(1 + r_{t+2})}) \quad (69)$$

Regarding borrowing constraints, individuals can be never constraint, which is the solution above. Otherwise, they can be constrained always or any combination of order of constrained and unconstrained periods. Exemplary, see here the solution for borrowing constraint in period 3 only:
For period 3:

$$a_{t+1} = a_{min} \quad (70)$$

and

$$I_t = \frac{K_t}{\Lambda_t(1 + K_t)}((1 + r_t)a_t + y_t - a_{min}) \quad (71)$$

For period 2, with $C = 1 + Kt + \beta_t(1 + K_{t+1})$:

$$a_{t+1} = \frac{\beta_t(1 + K_{t+1})}{C}((1 + r_t)a_t + y_t) - \frac{1 + K_t}{C}\frac{y_{t+1} - a_{min}}{1 + r_{t+1}} \quad (72)$$

$$I_t = \frac{K_t}{\Lambda_t C}((1 + r_t)a_t + y_t + \frac{y_{t+1} - a_{min}}{1 + r_{t+1}}) \quad (73)$$

For period 1, with $D = 1 + K_t + \beta_t(1 + K_{t+1} + \beta_{t+1}(1 + K_{t+2}))$:

$$a_{t+1} = \frac{\beta_t C}{D}((1 + r_t)a_t + y_t) - \frac{1 + K_t}{D}(\frac{y_{t+1}}{(1 + r_{t+1})} + \frac{y_{t+2} - a_{min}}{(1 + r_{t+1})(1 + r_{t+2})}) \quad (74)$$

$$I_t = \frac{K_t}{\Lambda_t D}((1 + r_t)a_t + y_t + \frac{y_{t+1}}{(1 + r_{t+1})} + \frac{y_{t+2} - a_{min}}{(1 + r_{t+1})(1 + r_{t+2})}) \quad (75)$$

Similar pathways can be constructed for households being borrowing constraint in period 2 and 1.

Optimal solution for c_t

If values for I_t, by that s_t and n_t, and a_{t+1} are determined, the optimal c_t simply is:

$$c_t = (1+r)a_t + y_t - p_{n,t}n_t - p_{s,t}s_t - a_{t+1} \tag{76}$$

Optimal solution if n_t is constrained

To derive the relative demands we take first-order conditions for the minimization problem:

$$\begin{aligned} &\min_{n_t,s_t} \quad p_{n,t}n_t + p_{s,t}s_t \\ &\text{s.t. } n_t \leq 5 \\ &\quad\quad It = [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} \end{aligned} \tag{77}$$

The Lagrangian looks the following:

$$\mathcal{L} = p_{n,t}n_t + p_{s,t}s_t - \lambda_{1,t}(I_t - [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}}) - \lambda_{2,t}(n_t - 5) \tag{78}$$

Deriving first order conditions in period 2 and 3:

$$\frac{\partial\mathcal{L}}{\partial s_t} = p_{s,t} - \lambda_{1,t}(a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t-1}\rho_t) \times [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}-1}\frac{1}{\rho_t} = 0 \tag{79}$$

$$\frac{\partial\mathcal{L}}{\partial n_t} = p_{n,t} - \lambda_{1,t}(n_t^{\rho_t-1}\rho_t) \times [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}-1}\frac{1}{\rho_t} - \lambda_{2,t} = 0 \tag{80}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_{1,t}} = I_t - [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + n_t^{\rho_t}]^{\frac{1}{\rho_t}} = 0 \tag{81}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_{2,t}} = n_t - 5 = 0 \tag{82}$$

If constraints are not binding, $\lambda_{2,t} = 0$, since $n_t < 5$. Then see solution above. If they are binding:

Taking ratios $\frac{\frac{\partial\mathcal{L}}{\partial n_t}}{\frac{\partial\mathcal{L}}{\partial s_t}}$ leads:

This means $n_t = 5$ and $I_t = [a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}$. If I_t is given, it follows:

$$s_t = \left(\frac{(I_t^{\rho_t} - 5^{\rho_t})}{a_{s,t}(Z_{s,t},\eta)}\right)^{\frac{1}{\rho_t}} \tag{83}$$

In case the household is constrained ($n_t = 5$), this price does not apply, as it uses the fact that, s_t can be expressed as a share of n_t given the level of investments. In the case that $n_t = 5$, therefore, the household maximizes differently (see next section). In period 1 $\Lambda_t = p_{n,t}$ as investment input decisions only take place for nutrition. This means $n_t = 5$ and $I_t = [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5]^{\frac{1}{\rho_t}}$

$$\begin{aligned}
V_t(Z_t, a_t, y_t, \Pi_t, \Psi_t) = \max_{c_t, s_t, a_{t+1}} & u(c_t) + \alpha_e v(\Psi_t) \\
& + \beta V_{t+1}(Z_{t+1}, a_{t+1}, y_{t+1}, \Pi_{t+1}, \Psi_{T+1}) \\
\text{s.t.} \quad c_t + 5p_{n,t} + p_{s,t}s_t + a_{t+1} &= (1+r)a_t + y_t \\
a_{t+1} &\geq a_{min,t} \\
\text{with} \quad \Psi_{t+1} &= \theta_t(Z_{\theta,t}) I_t^{\delta_{1,t}} \Psi_t^{\delta_{2,t}} \\
V_{T+1}(\Psi_{T+1}) &= \alpha_e \gamma_e \ln(\Psi_{T+1}) + \zeta \ln(a_{T+1}) \\
u(c_t) &= \ln(c_t) \\
v(\Psi_t) &= \ln(\Psi_t) \\
I_t &= [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}
\end{aligned} \tag{84}$$

Then:

$$\frac{\partial \mathcal{L}}{\partial s_t} = \beta \frac{\partial V_{t+1}}{\partial I_t} \frac{\partial I_t}{\partial s_t} - \lambda_t(p_{s,t}) = 0 \tag{85}$$

Drawing from the non-binding case, therefore:

$$\beta \frac{\partial V_{T+1}}{\partial I_t} = \frac{K_t}{I_t} = \frac{K_t}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}} \tag{86}$$

which results in:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial s_t} = \frac{K_t}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t}}} (a_{s,t}(Z_{s,t}, \eta)s_t^{(\rho_t - 1)} [a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]^{\frac{1}{\rho_t} - 1}) \\
- u'(c,t)p_{s_t} = 0
\end{aligned} \tag{87}$$

which yields:

$$u'(c,t)p_{s_t} = \frac{K_t}{[a_{s,t}(Z_{s,t}, \eta)s_t^{\rho_t} + 5^{\rho_t}]} a_{s,t}(Z_{s,t}, \eta)s_t^{(\rho_t - 1)} \tag{88}$$

Plugging in the budget constraint:

$$\frac{p_{s_t}}{-5p_{n_t} - p_{s_t}s_t - a_{t+1} + (1+r)a_t + y_t} = \frac{K_t}{[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}]}a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)} \quad (89)$$

yields:

$$0 = p_{s_t}[a_{s,t}(Z_{s,t},\eta)s_t^{\rho_t} + 5^{\rho_t}] - K_t a_{s,t}(Z_{s,t},\eta)s_t^{(\rho_t-1)}(-5p_{n_t} - p_{s_t}s_t - a_{t+1} + (1+r)a_t + y_t) \quad (90)$$

which can only be solved numerically.

GMM equations for investment parameters

To derive the relative demand ratios, one goes back to equation 26 and takes logs to get linear equations, using that $a_{s,t}(Z_{s,t},\eta) = exp(\phi_{s,t}Z_{s,t} + \eta)$:

$$\ln(\frac{p_{n,t}}{p_{s,t}}) = -\phi_{s,t}Z_{s,t} + (\rho_t - 1)\ln(\frac{n_t}{s_t}) - \eta$$
$$\ln(\frac{n_t}{s_t}) = \frac{1}{\rho_t - 1}Z'_{s,t}\phi_{s,t} - \frac{1}{1-\rho_t}\ln(\frac{p_{n,t}}{p_{s,t}}) - \frac{1}{1-\rho_t}\eta$$

Adding $\ln(\frac{p_{n,t}}{p_{s,t}})$ to both sides yields:

$$\ln(\frac{p_{n,t}n_t}{p_{s,t}s_t}) = \frac{1}{\rho_t - 1}Z'_{s,t}\phi_{s,t} + \frac{\rho_t}{\rho_t - 1}\ln(\frac{p_{n,t}}{p_{s,t}}) - \frac{1}{1-\rho_t}\eta$$

GMM equations for human capital parameters

$$\Psi_{t+1} = \theta_t(Z_{\theta,t})I_t^{\delta_{1,t}}\Psi_t^{\delta_{2,t}} \quad (91)$$

Using the human capital formation with $\theta_t(Z_{\theta,t}) = \exp(\phi_{\theta,t}Z_{\theta,t})$, taking logs:

$$\ln(\Psi_{t+1}) = \phi_{\theta,t}Z_{\theta,t} + \delta_{1,t}\ln(I_t) + \delta_{2,t}\ln(\Psi_t) \quad (92)$$

Since Ψ_t are latent skills, I assume the underlying measurement system with $S_{hs,t}$ and $S_{ts,t}$, which are observed height and test scores:

$$S_{ts_1,t} = \lambda_{ts_1,t} \ln(\Psi_t) + \epsilon_{ts_1,t} \tag{93}$$

and:

$$S_{ts_2,t} = \lambda_{ts_2,t} \ln(\Psi_t) + \epsilon_{ts_2,t} \tag{94}$$

Since height is observed in all periods, I can normalize $\lambda_{ts_1} = 1$ to allow for comparability of measures (see Cunha et al. (2010)).

Replacing the latent skills with the measurements leads too:

$$S_{ts_1,t+1} = \phi_{\theta,t} Z_t + \delta_{1,t} \ln(I_t) + \delta_{2,t} S_{ts_1} \tag{95}$$

and:

$$\frac{1}{\lambda_{ts_2,t+1}} S_{ts_2,t+1} = \phi_{\theta,t} Z_t + \delta_{1,t} \ln(I_t) + \delta_{2,t} \frac{1}{\lambda_{ts_2,t}} S_{ts_2} \tag{96}$$

To identify $\lambda_{ts_2,t}$ further equations are needed. To get these I exploit the covariance structure, similar to (Cunha et al., 2010). One can replace Ψ_t in equation 93 with using equation 93:

$$\frac{Cov(S_{ts_1,t}, S_{ts_1,t+1})}{Cov(S_{ts_2,t}, S_{ts_1,t+1})} = \lambda_{ts_2,t} \tag{97}$$

and:

$$\frac{Cov(S_{ts_1,t}, S_{ts_1,t+1})}{Cov(S_{ts_1,t}, S_{ts_2,t+1})} = \lambda_{ts_2,t+1} \tag{98}$$

Using that these measures have mean 0, the covariance can be rearanged to:

$$0 = \mathrm{E}[(S_{ts_1,t+1} - \lambda_{ts_2,t+1} S_{ts_2,t+1}) S_{ts_1,t}] \tag{99}$$

and:

$$0 = \mathrm{E}[(S_{ts_1,t} S - \lambda_{ts_2,t} S_{ts_2,t}) S_{ts_1,t+1}] \tag{100}$$

A.7 Additional tables

TABLE A.7: Distribution of parenting skill types η by total amount of types

	Observations for type:				
Amount of types	Type 0	Type 1	Type 2	Type 3	Type 4
K=2	2,020	4,417			
K=3	2,992	2,831	614		
K=4	2,813	2,956	391	277	
K=5	2,664	547	2,863	9	354

Note: This table summarizes the amount of observation for each set of types, for different total amount of types secified.

TABLE A.8: Investment gap decomposition by childhood period

	Investment gap (%):		
	Early childhood	Primary school	High school
Baseline gap	12.38	12.07	6.77
Closing the gap by:			
Preferences	86.13	91.72	89.74
+ Investment productivities	86.13	105.13	131.47
+ Skill productivities	86.13	105.13	131.47
+ Income	16.28	14.81	16.17
+ Assets	1.22	-1.20	-1.27

Note: Gap indicated are between high school parents and parents with no schooling. Rest of the gap derives from differences in initial skills and prices and survey year.

Table A.9: Transmission of effects into adult skills

Period of change:	Adult skills increase (std.) Early childhood	Primary school	High school
Closing differences in:			
Nutrition	0.0013	0.0040	0.0112
Schooling	0.0000	0.0003	0.0076
Increase by 0.1SD:			
Current skills	0.0004	0.0041	0.0216

Note: Values calculated for average parents with no schooling. For investment offsetting avergage investments of high school parents are taken.

Table A.10: Policy counterfactuals - investment change

	Cash transfer	Nutrition subsidy	Schooling subsidy	Cash+ nutrition	Cash+ schooling	Nutrition+ schooling
Change in mean investments (%):						
Primary school	1.21	16.69	4.29	17.79	5.25	20.49
High school	1.43	15.75	13.26	17.40	14.82	32.08

Note: Policies are designed to have the same costs (in 100,000 rupees $\sim$ \$7), resulting in a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy.

Table A.11: Policy counterfactuals by income decile

Income decile:	1	2	3	4	5	6	7	8	9	10
Change in mean skills (SD):										
Cash	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Nutrition	0.04	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02	0.01
Schooling	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02
Change in mean investements (%):										
Cash	1.74	1.01	0.69	0.71	0.61	0.75	0.34	0.37	0.24	0.03
Nutrition	17.91	14.89	13.18	13.46	11.72	10.93	10.28	9.01	9.10	5.35
Schooling	8.19	9.47	7.83	8.83	8.12	8.93	8.93	9.32	9.36	6.78
Cost by 0.01 SD increase:										
Cash	1.45	2.83	2.78	3.72	2.99	2.86	8.53	7.27	14.55	51.95
Nutrition	0.17	0.25	0.30	0.34	0.46	0.37	0.51	0.63	0.71	1.29
Schooling	0.24	0.32	0.44	0.57	0.63	0.67	0.89	1.00	1.38	3.92

Note: Costs are expressed in 100,000,000 rupees ($\sim$ \$0,007), simulated are a 3% cash transfer, 20% nutrition subsidy and 99% schooling subsidy.

Table A.12: Estimation results for skill formation parameters for 2 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.10	(0.65)***	-10.12	(4.16)**
Implied elasticity			0.24		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-2.24	(0.39)***	-35.08	(12.33)***
Mother primary			0.88	(0.19)***	2.58	(1.02)**
Mother high			1.51	(0.30)***	4.14	(1.62)**
Father primary			0.01	(0.14)	0.38	(0.38)
Father high			-0.18	(0.17)	0.20	(0.41)
Age			-0.04	(0.04)	2.80	(1.05)***
Female			0.06	(0.11)	1.21	(0.52)**
Rural area			-2.27	(0.41)***	-4.47	(1.74)**
No. of siblings			-0.61	(0.11)***	-1.90	(0.71)***
Mother not Islam			0.32	(0.19)*	1.35	(0.67)**
Parenting type 1			-1.53	(0.29)***	-3.13	(1.23)**
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.23	(0.03)***
$\delta_{2,t}$ (skills)	0.08	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.26	(0.10)***
Mother primary	0.06	(0.04)	0.08	(0.04)*	0.05	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.14	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.97	(0.11)	1.06	(0.01)	1.12	(0.01)
$\lambda_{4,ts}$					1.26	(0.04)
Observations	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

Table A.13: Estimation results for skill formation parameters for 3 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.37	(0.74)***	-10.37	(4.36)**
Implied elasticity			0.23		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.68	(0.47)***	-38.98	(14.12)***
Mother primary			1.01	(0.22)***	2.84	(1.14)**
Mother high			1.71	(0.34)***	4.54	(1.81)**
Father primary			0.05	(0.15)	0.51	(0.41)
Father high			-0.12	(0.17)	0.37	(0.44)
Age			-0.05	(0.04)	2.89	(1.11)***
Female			0.03	(0.12)	1.19	(0.53)**
Rural area			-2.44	(0.46)***	-4.75	(1.89)**
No. of siblings			-0.67	(0.12)***	-1.98	(0.76)***
Mother not Islam			0.33	(0.20)	1.43	(0.71)**
Parenting type 1			0.14	(0.13)	0.03	(0.31)
Parenting type 2			3.49	(0.68)***	6.45	(2.55)**
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.22	(0.03)***
$\delta_{2,t}$ (skills)	0.08	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.25	(0.09)***
Mother primary	0.06	(0.04)	0.08	(0.04)*	0.05	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.14	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)*	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.97	(0.11)	1.06	(0.01)	1.13	(0.01)
$\lambda_{4,ts}$					1.26	(0.04)
Observations	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.

TABLE A.14: Estimation results for skill formation parameters for 5 types

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.19	(0.68)***	-9.81	(3.92)**
Implied elasticity			0.24		0.09	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.61	(0.44)***	-37.26	(12.76)***
Mother primary			0.98	(0.21)***	2.71	(1.04)***
Mother high			1.61	(0.31)***	4.39	(1.66)***
Father primary			0.06	(0.14)	0.54	(0.40)
Father high			-0.12	(0.17)	0.39	(0.42)
Age			-0.04	(0.04)	2.75	(1.00)***
Female			0.04	(0.11)	1.10	(0.48)**
Rural area			-2.37	(0.43)***	-4.63	(1.74)***
No. of siblings			-0.64	(0.11)***	-1.89	(0.69)***
Mother not Islam			0.36	(0.19)*	1.41	(0.67)**
Parenting type 1			1.52	(0.35)***	2.32	(1.01)**
Parenting type 2			-0.04	(0.12)	0.36	(0.33)
Parenting type 3			-0.04	(2.44)	16.02	(7.23)**
Parenting type 4			4.25	(0.82)***	8.36	(3.17)***
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.07)***	0.14	(0.06)**	0.23	(0.03)***
$\delta_{2,t}$ (skills)	0.07	(0.03)**	0.18	(0.02)***	0.20	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.75	(0.10)***	-0.06	(0.13)	-0.27	(0.09)***
Mother primary	0.06	(0.04)	0.07	(0.04)*	0.06	(0.03)**
Mother high	0.24	(0.04)***	0.26	(0.05)***	0.13	(0.03)***
Father primary	-0.01	(0.05)	0.13	(0.04)***	0.04	(0.03)
Father high	0.08	(0.04)**	0.08	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)*	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	0.98	(0.11)	1.07	(0.01)	1.13	(0.01)
$\lambda_{4,ts}$					1.27	(0.04)
Observations	27,257					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1.

Table A.15: Robustness check: GMM without constrained individuals

	Early childhood		Primary school		High school	
Investment elasticity:						
Substitution parameter ρ_t			-3.33	(0.76)***	-14.60	(8.71)*
Implied elasticity			0.23		0.06	
Schooling investment productivity $\phi_{s,t}$:						
Constant			-3.42	(0.48)***	-53.02	(28.45)*
Mother primary			1.10	(0.24)***	4.10	(2.34)*
Mother high			1.78	(0.37)***	7.24	(4.12)*
Father primary			0.23	(0.16)	0.79	(0.69)
Father high			0.05	(0.19)	0.25	(0.62)
Age			-0.04	(0.04)	4.05	(2.26)*
Female			0.02	(0.13)	1.53	(0.94)
Rural area			-2.36	(0.46)***	-6.64	(3.80)*
No. of siblings			-0.68	(0.13)***	-2.71	(1.51)*
Mother not Islam			0.21	(0.21)	2.06	(1.34)
Parenting type 1			-0.37	(0.15)**	-0.68	(0.56)
Parenting type 2			4.26	(0.90)***	12.26	(6.99)*
Parenting type 3			1.62	(0.52)***	2.93	(1.99)
Human capital formation:						
$\delta_{1,t}$ (investment)	0.28	(0.06)***	0.16	(0.05)***	0.17	(0.03)***
$\delta_{2,t}$ (skills)	0.10	(0.02)***	0.19	(0.02)***	0.22	(0.01)***
Total factor productivity $\phi_{\theta,t}$:						
Constant	-0.73	(0.08)***	-0.02	(0.12)	-0.21	(0.09)**
Mother primary	0.02	(0.04)	0.06	(0.04)	0.05	(0.02)**
Mother high	0.22	(0.03)***	0.25	(0.04)***	0.16	(0.03)***
Father primary	0.02	(0.04)	0.13	(0.04)***	0.03	(0.03)
Father high	0.11	(0.03)***	0.07	(0.04)*	0.10	(0.03)***
Age	0.10	(0.01)***	-0.02	(0.01)**	0.01	(0.01)
Factor loading for skills $\lambda_{ts_2,s}$:						
$\lambda_{t,ts}$	1.00	(0.07)	1.07	(0.01)	1.09	(0.01)
$\lambda_{4,ts}$					1.21	(0.04)
Observations	27,366					

Note: Standard errors in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. All coefficients are from a single GMM estimation.