

Working Paper Series

**Multifactor Inequality: Substitution Effects for  
Income Sources in Mexico**

Maria Elena Garcia Reyes

**ECINEQ WP 2006 – 31**

## **Multifactor Inequality: Substitution Effects for Income Sources in Mexico**

**Maria Elena García Reyes\***

*University of York*

May 31, 2005

### **Abstract**

Income is a poor indicator of welfare and should be use only as a as component of welfare. Wealth provides another dimension of well-being. We use the Kullback-Leibler Information Criterion (KLIC, hereafter) as a measure of the discrepancy between two attributes. This will allow economists and researchers to understand the kind of relationship that the attributes hold. The applications deals with an unusual analysis of income factors in which we treat them as different attributes. This allows us to estimate the substitution effects among income factors.

Keywords: entropy, cross-entropy, KLIC, multi-attribute, multifactor, welfare matrix, Theil index, Gini index

JEL Classification: D33

---

\* **Contact details:** Maria Elena Garcia Reyes; University of York; Department of Economics; Heslington, York YO105DD, U.K. [megr100@york.ac.uk](mailto:megr100@york.ac.uk)

## 1 Introduction

Welfare distribution has widely been discussed as the distribution of income. This means that individuals are not distinguished by anything other than income, because their other differences are either not known or are considered as irrelevant, or a mixture of both.<sup>1</sup> There are several reasons to believe that income is a poor indicator of welfare and should be used only as a component of welfare. Wealth provides another dimension of well-being. Two people who have the same income may not be equally well off if one person has more wealth. If one person owns his home, for example, and the other person doesn't, then the former is better off. Many other attributes that compose an individual's well-being such as consumption, education, health or other attributes, are relevant to the individual's well-being. In the literature there has been an increasing interest in the field of multidimensional inequality. First by Kolm [7] followed

---

<sup>1</sup>See Kolm [7].

by Atkinson and Bourguignon [2] who explored the foundation of multidimensional inequality. In the recent years there has been expanding literature on the topic<sup>2</sup>.

We believe that the sources of income are dependent. Furthermore, we believe that the sources of income compensate each other as a response to inequality and/or polarisation. In other words, when one income source diminishes another increases to compensate for either inequality or polarisation. Therefore, it is important to measure the degree of mutual information between the sources of income as an approximation of early stages of polarisation.

The Kullback-Leibler Information Criterion (KLIC, hereafter) is a measure of the discrepancy between two probability distributions. In our case between two sources of income. The KLIC ( $KL(v_j, v_i)$ ) is also referred to as the cross-entropy of  $v_j$  relative to  $v_i$ . In the discrete case it is defined by:

$$KL(v_j, v_i) = \sum_k v_j \ln(v_j/v_i) \quad \text{for } i \neq j \quad (1)$$

where:

$v_j, v_i$  denote the finite or countable vectors of probability weights implied by the discrete distributions.

$i = (1, 2, \dots, m)$

$j = (1, 2, \dots, m)$

$m = 6$  : sources of income

$v_1$  :Probability of receiving earned income:  $y_1$

$v_2$  :Probability of receiving self employed income:  $y_2$

$v_3$  :Probability of receiving property income:  $y_3$

$v_4$  :Probability of receiving income derived from cooperatives:  $y_4$

$v_5$  :Probability of receiving employers contributions:  $y_5$

$v_6$  :Probability of receiving other sources of income:  $y_6$

The distribution in the second position of the argument list in  $KL(v_j, v_i)$ , here  $v_i$ , is called the reference distribution. We refer to the first distribution in the argument list, here  $v_j$  as the subject distribution. In terms of our research the KLIC  $KL(v_j, v_i)$  is the cross-entropy of income source  $j$  relative to income source  $i$ . In other words, the KLIC between two distributions is the response of income source  $j$  relative to the subject distribution of income source  $i$ .

Properties

- KL is not symmetric in  $v_j$  and  $v_i$  and thus in general  $KL(v_j, v_i) \neq KL(v_i, v_j)$
- $KL(v_j, v_i) > 0$  for every  $v_j > 0$  and
- $KL(v_j, v_i) = 0$  if  $v_j = v_i$ .
- The greater the value of  $KL(v_j, v_i)$ , the greater the discrepancy between the distributions  $v_j$  and  $v_i$ .

---

<sup>2</sup>See Maasoumi [9], Tsiu [17].

From the fact that the KLIC is not symmetric, note that it is not a true measure of the distance between  $v_j$  and  $v_i$  and it is referred to instead as a pseudo-distance function. For our purposes it is very important to notice this characteristic since one could easily misinterpret the results. Therefore, this characteristic should be borne in mind and when reporting results, all pairs of KLIC must be interpreted the resulting matrix will not be symmetric.

The KLIC will not only exhibit discrepancy between income factors but also measure how income factors compensate each other.

In terms of our hypothesis, this means that if there is no increased inequality or polarisation then the values for the KLIC will remain the same:

$$KL_t(v_j, v_i) = KL_{t+1}(v_j, v_i)$$

It is only in the presence of inequality or polarisation that the income factors respond to these changes and the KLIC will differ:

$$KL_t(v_j, v_i) \neq KL_{t+1}(v_j, v_i)$$

These attributes makes the KLIC a very interesting measure of polarisation. The results are presented on the following matrix for each year that has been analysed.

Table 1: Kullback-Leibler Information Criterion Matrix

$$\left( \begin{array}{cccccc} 0 & KL(v_2, v_1) & KL(v_3, v_1) & KL(v_4, v_1) & KL(v_5, v_1) & KL(v_6, v_1) \\ KL(v_1, v_2) & 0 & KL(v_3, v_2) & KL(v_4, v_2) & KL(v_5, v_2) & KL(v_6, v_2) \\ KL(v_1, v_3) & KL(v_2, v_3) & 0 & KL(v_4, v_3) & KL(v_5, v_3) & KL(v_6, v_3) \\ KL(v_1, v_4) & KL(v_2, v_4) & KL(v_3, v_4) & 0 & KL(v_5, v_4) & KL(v_6, v_4) \\ KL(v_1, v_5) & KL(v_2, v_5) & KL(v_3, v_5) & KL(v_4, v_5) & 0 & KL(v_6, v_5) \\ KL(v_1, v_6) & KL(v_2, v_6) & KL(v_3, v_6) & KL(v_4, v_6) & KL(v_5, v_6) & 0 \end{array} \right)$$

Note that the main diagonal is equal to zero, and that the upper and lower diagonals are different. Therefore the matrix is not symmetric. Thus the Kullback-Leibler Information Criterion (KLIC, hereafter) which is a measure of divergence or distance between distributions. We aim to incorporate our application and other properties of the KLIC to the literature of multidimensional indices. The KLIC is a measure of the discrepancy between two probability distributions, in our case the distributions of two attributes. This will allow economists and researchers to understand the kind of relationship that the attributes hold. By this we mean that attributes may be complement or substitute goods between them.

In this paper we discuss the main definitions of multidimensional inequality, and the way the KLIC fits into the literature. Only two main applications were derived, multifactor analysis for the Mexican case and multidimensional inequality in Mexico. The first application deals with an usual analysis of income factors in which we treat them as different attributes. This allows us to estimate the substitution effects among income factors. The second application

deals with the estimation of multidimensional inequality in Mexico using two attributes: monetary and non monetary income. Both applications seemed to have very intuitive results. Unfortunately our data base does not cover other variables apart from monetary and non monetary income and thus the impossibility of estimating multiattribute inequality. And although we do not have data for other attributes, different from income, the procedure of estimation would remain the same. It is kept for further research to obtain non income data and estimate multiattribute inequality using this procedure.

## 2 Background and Definitions

Let us start by defining the welfare matrix  $y$  as:

$$y = \begin{pmatrix} y_{11} & y_{12} & y_{13} & \cdot & \cdot & \cdot & y_{1m} \\ y_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & y_{2m} \\ y_{31} & \cdot & \cdot & \cdot & \cdot & \cdot & y_{3m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ y_{n1} & y_{n2} & \cdot & \cdot & \cdot & \cdot & y_{nm} \end{pmatrix} \quad (2)$$

where:

- $y_{ij}$  be the amount of attribute  $m$  received by individual or household  $i$ .
- $i = 1, \dots, n$  : the number of individuals or households and,
- $j = 1, \dots, m$  : the attributes.

Thus, the Welfare Matrix  $y$  (2) is a matrix of size  $n \times m$ . The Welfare Matrix is associated with the Probability Welfare Matrix  $v$  (3) in that it is a matrix of size  $n \times m$  and each element  $v_{ij}$  is interpreted as the probability of individual  $i$  of receiving  $j$  type of income<sup>3</sup>

$$v = \begin{pmatrix} v_{11} & v_{12} & v_{13} & \cdot & \cdot & \cdot & v_{1m} \\ v_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & v_{2m} \\ v_{31} & \cdot & \cdot & \cdot & \cdot & \cdot & v_{3m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ v_{n1} & v_{n2} & \cdot & \cdot & \cdot & \cdot & v_{nm} \end{pmatrix} \quad (3)$$

The two Theil indices which belong to the Generalized Entropy Family (GE hereafter) also used in Chapter 3 are:

$$I_0(S) = \sum_{i=1}^N S_i^* \ln(S_i^*/p_i) \quad (4)$$

$$I_1(S) = \sum_{i=1}^N p_i \ln p_i/S_i^* \quad (5)$$

where:

$$S_i^* = S_i / \sum_j S_j$$

---

<sup>3</sup>  $\sum_i x_{im} = 1$

$p_i = 1/n$  is the units population share.

The GE are homogenous (in a single attribute), symmetric, and consistent with the Lorenz criterion with respect to  $S$ . Useful decomposability properties of  $I(\cdot)$ .

According to Maasouni the multidimensional inequality index for one of Theil measure is:

$$D_{-1}(v_{if}, S^*; Cf) = \sum_f C_f \left( \sum_i v_{if} \ln \frac{v_{if}}{S_i^*} \right) \quad (6)$$

where:

$$\begin{aligned} S_i^* &= \sum_f \delta_f v_{if} / K \\ K &= \sum_f \delta_f T_f \\ C_f &= \delta_f T_f / \sum_k \delta_k T_k \\ T_f &:= \sum_i v_{if} \\ \delta_f &= \alpha_f / \sum_f v_f \end{aligned}$$

$$D_0(S^*, x_{if}; \delta_f) = \sum_f \delta_f \left( \sum_i S_i^* v_{if} \ln \frac{S_i^*}{v_{if}} \right) \quad (7)$$

where:

$$\begin{aligned} S_i^* &= \prod_{j=1}^f v_{if}^{\delta_j} / K \\ K &= \sum_i \left( \prod_f x_{if}^{\delta_f} \right) \end{aligned}$$

Both (6) and (7) are weighted averages of the KLIC. It follows that multidimensional inequality is no more than the weighted average of attribute inequalities and that this reflects the substitution effects between sources of income. Theil's second measure (7) satisfies "general homogeneity" property and Symmetry.

Two main applications stand out. The first is linked with our last chapter and some of the discussion of income sources. We can now estimate the substitution effects between income sources. The second application is regarding the multi-attribute case. Estimating multidimensional inequality incorporating other attributes such as monetary and non monetary income for the Mexican Case.

In both cases the application will use Multidimensional two Theil Index (7) because it satisfies the properties of "general homogeneity" property and symmetry.

### 3 Multifactor Inequality in Mexico and Income Sources Substitution Effects

The Probability Welfare Matrix  $v$  (3) that is a matrix of size  $n \times m$  and each element  $v_{ij}$  is interpreted as the probability of individual  $i$  of receiving  $j$  type

of income<sup>4</sup>

$$v = \begin{pmatrix} v_{11} & v_{12} & v_{13} & \cdot & \cdot & \cdot & v_{1m} \\ v_{21} & \cdot & \cdot & \cdot & \cdot & \cdot & v_{2m} \\ v_{31} & \cdot & \cdot & \cdot & \cdot & \cdot & v_{3m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ v_{n1} & v_{n2} & \cdot & \cdot & \cdot & \cdot & v_{nm} \end{pmatrix} \quad (8)$$

where:

$$i = (1, 2, \dots, n)$$

$$f = (1, 2, \dots, m)$$

$n$  : The number of households.

$m = 6$  : Sources of income

$y_{i,1}$  : Earned income,

$y_{i,2}$  : Self employed income,

$y_{i,3}$  : Property income,

$y_{i,4}$  : Income derived from cooperatives,

$y_{i,5}$  : Employers contributions and,

$y_{i,6}$  : Other sources of income

$$D_0(S^*, v_{im}; \delta_f) = \sum_f \delta_f \left( \sum_i S_i^* v_{im} \ln \frac{S_i^*}{v_{im}} \right) \quad (9)$$

where:

$$S_i^* = \prod_{f=1}^M v_{im} / K$$

$$K = \sum_i \left( \prod_f v_{im} \right)$$

$$\delta_f = \alpha_f / \sum_f \alpha_f$$

The results obtained were:

Table 2: Multifactor Inequality

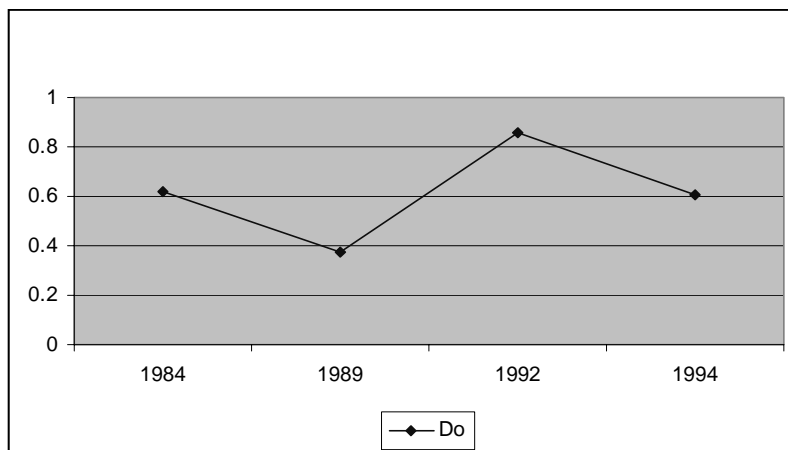
| $D_0$ |             |
|-------|-------------|
| 1984  | 0.618946669 |
| 1989  | 0.371157319 |
| 1992  | 0.858622255 |
| 1994  | 0.603259151 |

---

<sup>4</sup>  $\sum_i v_{im} = 1$



Figure 1: Multifactor Inequality in Mexico



The values of the Gini Index of Relative Inequality using the covariance approach (??) are shown in the following table:

Table 3: Gini Index

| Year | GINI Index | Rank |
|------|------------|------|
| 1984 | 0.51224303 | 3    |
| 1989 | 0.50954235 | 2    |
| 1992 | 0.57664616 | 4    |
| 1994 | 0.48451204 | 1    |

Notice that the results are consistent in trend with those in *Table 3*) but they differ in magnitude. Let's recall Maasoumi's (1986) [9] interpretation of multidimensional inequality. "Multidimensional inequality is no more than the weighted average of attribute inequalities. This reflects the substitution effects<sup>5</sup> of the income factors shown on *Table (7)*.

The above table not only makes sense but is also consistent with the estimations on Chapter 3 (*See Table (3.1)*). This approach has allowed us to put together most of the results related to income factor analysis and to obtain a value of the substitution effects between factors. The substitution effects are, in our opinion, very valuable estimates because they express how much one source compensates for existing inequality and it also reflects early stages of polarisation.

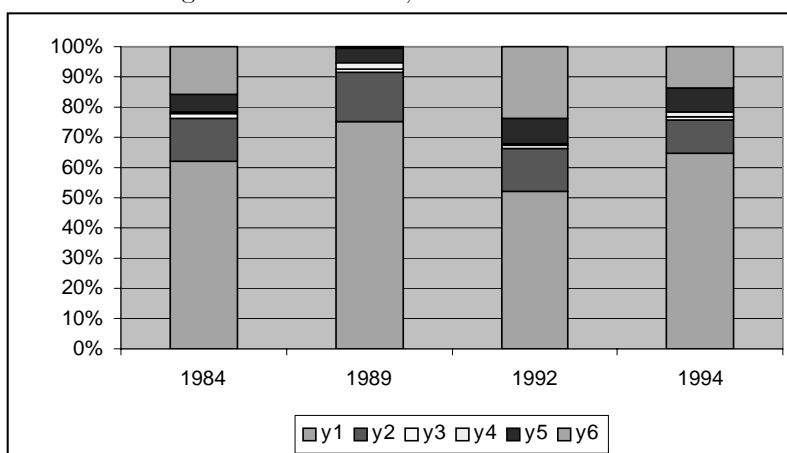
The next graph shows the substitution effects of the income factors using a multifactor analysis.

<sup>5</sup>Substitution Effect: when the price of a good increases it does so relative to all other goods. Although each good is unique it has substitutes - other goods that will serve almost as well. As the opportunity cost of a good rises, people will tend to buy less of it and more of its substitutes.

Table 4: Substitution Effects between Income Factors

|       | 1984        | 1989        | 1992        | 1994        |
|-------|-------------|-------------|-------------|-------------|
| $y_1$ | 0.384594786 | 0.280250495 | 0.448665933 | 0.390663415 |
| $y_2$ | 0.087517449 | 0.060059978 | 0.119302945 | 0.065346989 |
| $y_3$ | 0.0106158   | 0.004008086 | 0.012182658 | 0.006274994 |
| $y_4$ | 0.00103653  | 0.008084923 | 0.002560909 | 0.009824298 |
| $y_5$ | 0.038089643 | 0.016918159 | 0.072060755 | 0.049327871 |
| $y_6$ | 0.097092461 | 0.001835677 | 0.203849055 | 0.081821585 |

Figure 2: Multifactor, Substitution Effects



We can see that Earned income ( $y_{i,2}$ ) and, Self employed income ( $y_{i,2}$ ) are good substitutes and this is quite reasonable, since most of your income is derived from labour in one way or another. Another income factor with high substitution effect is ( $y_{i,6}$ ) Other sources of income. This factor contains income from the sale of second-hand items, such as selling cars, heritage, lottery, loans, etc. The other values are smaller which means that the other income sources act as complementary goods rather than substitute goods. In the presence of income inequality and/or polarisation, agents will first compensate for it by increasing those income factors with higher substitution effect, rather than the ones that are complements.

### 3.1 Standard Theory: Decomposition of Inequality by Income Factors

Most index numbers in common usage exhibit some kind of decomposition property that enables the overall index value to be computed from sub-aggregates. These sub-aggregates are typically based on grouping together observations which share common characteristics.

The problem can be formally stated if we denote the income of an individual  $i$  from source  $j$  by  $y_{ij}$  ( $j = 1, 2, \dots, m$ ) for  $m$  types of income. The distribution of factor  $j$  is represented by  $(y_{1j}, \dots, y_{nj}) = y_j$ . Assuming that the factor income categories are mutually exclusive, and that the total income is the sum of individual factor incomes, we have

$$y_i = \sum_{j=1}^m y_{ij}$$

Thus, the assessment of inequality contribution of factor  $j$  becomes the problem of determining the impact of  $y_j$  on  $y_i$ . If for any inequality index  $I(y_i)$  we have some way of writing  $I(y_i) = \sum_{j=1}^n \beta_j$ , where  $\beta_j$  depends on income from source  $j$ , then  $\beta_j$  might be regarded as the contribution of factor  $j$  to aggregate inequality. Equivalently, the fractions  $b_j = \frac{\beta_j}{I(y_i)}$  can be interpreted as proportional factor contributions.

We say that a function is a decomposition rule if it produces suitable values of  $b_j$  with the following properties:

$$\begin{aligned} \sum b_j &= 1 \\ \sum \beta_j &= I(y_i) \end{aligned}$$

For the Gini Index the decomposition rule is :

$$b_j = \frac{\sum_{i=1}^n [i - \frac{n+1}{2}] \bar{y}_j}{\sum_{i=1}^n [i - \frac{n+1}{2}] \bar{y}_i} \quad (10)$$

where:

$$\begin{aligned} \sum b_j &= 1 \\ \sum \beta_j &= GINI \end{aligned}$$

The following results were obtained:

Table 5: Factor Contributions to Inequality

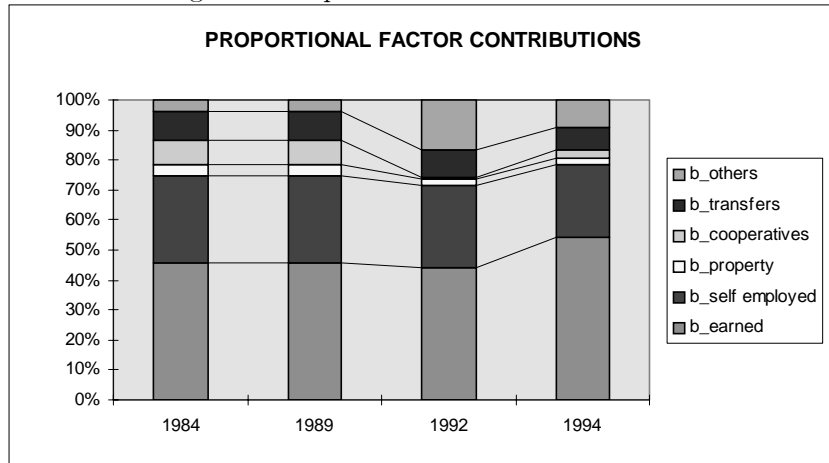
|                          | 1984      | 1989          | 1992          | 1994          |
|--------------------------|-----------|---------------|---------------|---------------|
| $\beta_{EARNED}$         | 0.2365332 | 0.2291129571  | 0.22402225523 | 0.26451417461 |
| $\beta_{SELF\_EMPLOYED}$ | 0.1339064 | 0.14888827467 | 0.1718982203  | 0.11521696311 |
| $\beta_{PROPERTY}$       | 0.025268  | 0.02109505329 | 0.01228256321 | 0.00935108237 |
| $\beta_{COOPERATIVES}$   | 0.0011633 | 0.04162961    | 0.00351754158 | 0.01366323953 |
| $\beta_{TRANSFERS}$      | 0.0523992 | 0.04957847066 | 0.05708796984 | 0.03493331808 |
| $\beta_{OTHERS}$         | 0.0629652 | 0.01926070083 | 0.10431529034 | 0.04278241313 |
| $\sum \beta_i = Gini$    | 0.5122354 | 0.50956506654 | 0.5731238405  | 0.48046119084 |

Table 6: Proportional Factor Contributions to Inequality

|                      | 1984   | 1989   | 1992   | 1994   |
|----------------------|--------|--------|--------|--------|
| $b_{EARNED}$         | 0.4736 | 0.4677 | 0.4752 | 0.5253 |
| $b_{SELF\_EMPLOYED}$ | 0.2613 | 0.2922 | 0.2981 | 0.2378 |
| $b_{PROPERTY}$       | 0.0493 | 0.0414 | 0.0213 | 0.0190 |
| $b_{COOPERATIVES}$   | 0.0023 | 0.0817 | 0.0061 | 0.0282 |
| $b_{TRANSFERS}$      | 0.1023 | 0.0973 | 0.0990 | 0.0721 |
| $b_{OTHERS}$         | 0.1229 | 0.0378 | 0.1809 | 0.0883 |
| $\sum b_i$           | 1      | 1      | 1      | 1      |

It is not surprising that the highest factor of contribution is due to what can be described as payment for any kind of labour, including earned income and self-employed income, but there is one result that is worthy of more attention. This is the fact that the rest of the factor contributions reversed their positions in 1992.

Figure 3: Proportional Factor Contributions



## 4 Multidimensional Inequality in Mexico

For this application we will define welfare the matrix as  $W$ :

$$W = \begin{pmatrix} w_{11} & w_{1,2} \\ w_{2,1} & w_{22} \\ w_{31} & w_{32} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ w_{n1} & w_{n2} \end{pmatrix} \quad (11)$$

where:

$$i = (1, 2, \dots, n)$$

$$f = (1, 2)$$

$n$  : The number of households.

$m = 2$  There are two attributes monetary income and non monetary income.

$$D_0(S^*, v_{im}; \delta_f) = \sum_f \delta_f \left( \sum_i S_i^* v_{im} \log \frac{S_i^*}{v_{im}} \right) \quad (12)$$

where:

$$S_i^* = \prod_{f=1}^M v_{im}/K$$

$$K = \sum_i (\prod_f v_{im})$$

$$\delta_f = \alpha_f / \sum_f \alpha_f$$

The results obtained were:

Table 7: Multidimensional Inequality

| $D_0$ |             |
|-------|-------------|
| 1984  | 0.626453124 |
| 1989  | 0.603724627 |
| 1992  | 0.638633253 |
| 1994  | 0.59023655  |

These results are consistent with the Gini Coefficient (*See Table ??*) and with the previous results they differ in magnitude but they follow the same trend. The difference in magnitude should be a topic of further research.

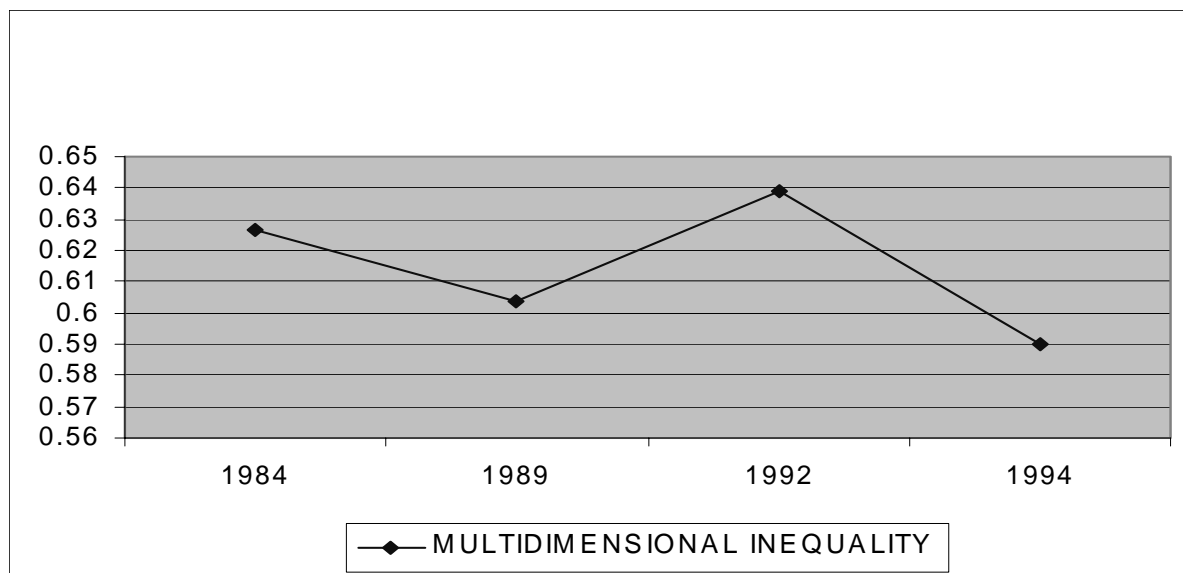
The substitution effects are shown on the following table:

Table 8: Substitution Effects between Monetary and Non-Monetary Income

|                     | 1984        | 1989        | 1992        | 1994        |
|---------------------|-------------|-------------|-------------|-------------|
| Monetary            | 0.451266179 | 0.430240301 | 0.497773814 | 0.372933906 |
| Non-Monetary Income | 0.175186945 | 0.173484326 | 0.140859439 | 0.217302643 |

The substitution effects between monetary and non-monetary income are consistent with intuition. Monetary income has a higher substitution effect.

Figure 4: Multidimensional Inequality



## 5 Conclusions

Two main applications of multidimensional inequality were drawn in this chapter. The first is regarding multifactor income inequality, maybe the most interesting application for the purposes of the research. This concluding chapter manages to put together consistently the different estimations for income factor contribution to overall inequality into one single index with the most useful interpretation of the substitution effects among factors.

There are several ways to express the degree of income inequality in a society, we have chosen to decompose income and the participation of each income factor into overall income inequality. This technique is relatively simple and revealing; it has allowed us to understand not only the impact of each income source to overall inequality but also it reveals the relationship between those income factors. When we first decompose inequality into income factors we did it believing there was a relationship between income factors and that those were not independent. (*See Table ??*) We used standard theory to describe the participation of each income source into overall inequality. Those estimations made us suspect about the relationship among income factors and the final results presented in this chapter showed that the relationship between income factors may be expressed in terms of their substitution effects. In practical terms it means that policy makers not only have, more information but a more powerful one. Policies designed to decrease income inequality may do so, by increasing by any means income derived from sources that act as complementary “goods”. Those income sources that reflect payment to labour have higher sub-

stitution effects the other sources of income may be regarded as complementary “goods”. The other application deals with multi-attribute inequality, applied to monetary and non monetary income. Monetary income is able to substitute non monetary income in a higher proportion than non monetary income. The applications of multi-attribute inequality have a wide range. Attributes such as education, health and any other attribute that contributes to the individual well being.

## References

- [1] Abadir K. and Magnus J. (2002). Notation in econometrics: a proposal for a standard. *Econometrics Journal* 5:76-90.
- [2] Atkinson A. and Bourguignon F. (1982). The Comparison of Multi-Dimensioned Distributions of Economic Status. *The Review of Economic Studies* 49, 2:183-201.
- [3] Charkravarty S. (1990). *Ethical Social Index Numbers*. Berlin, Springer.
- [4] Cardoso J. (1998). Multidimensional independent component analysis, Proc. ICASSP'98.
- [5] Duclos J., Sahn D. and Younger S. (2003). Robust Multidimensional Poverty Comparisons, Université Laval - Département d'économique in its series Cahiers de recherche, number 0115. and Taber C. (2000). Direct Estimation of Policy Impacts NBER Working Paper No. 0254.
- [6] INEGI (1984, 1989, 1992 and 1994). Methodological Syntheses of The National Survey of Income and Expenditure of Households. (Sintesis Metodologica de la Ecuesta Nacional de Ingreso Gasto de los Hogares).
- [7] Kolm S. (1977). Multidimensional Egalitarianisms. *The Quarterly Journal of Economics*, 91 1:1-13.
- [8] Lambert P. (1993) and (2000) *The Distribution and Redistribution of Income*, Manchester University Press. ISBN: 0719057329.
- [9] Maasoumi E. (1986). The Measurement and Decomposition of Multi-Dimensional Inequality. *Econometrica*, 54 4:991-998.
- [10] Shorrocks A. (1980). The Class of Additively Decomposable Inequality Measures. *Econometrica*, 48 3:613-625.
- [11] Shorrocks A. (1982). Inequality Decomposition by Factor Components. *Econometrica* 50 1:193-211.
- [12] Shorrocks A. and Mookherjee D. (1982). A Decomposition Analysis of the Trend in UK Income Inequality. *The Economic Journal*, 92 368:886-902.
- [13] Shorrocks A. (1984) Inequality Decomposition by Population Subgroups. *Econometrica* 52 6:1369-1385.
- [14] Shorrocks A. and Davies J. (1989). Optimal Grouping of Income and Wealth Data. *Journal of Econometrics* 42:97-108.
- [15] Shorrocks A. (1999). Decomposition procedures for distributional analysis: a unified framework based on the Shapley value'. Department of Economics, University of Essex, mimeo.



- [16] Theil H. (1967) *Economics and Information Theory*.
- [17] Tsui K. (1999). Multidimensional inequality and multidimensional generalized entropy measures: An axiomatic derivation. *Social Choice and Welfare*16:145-157.